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Alberta Electric System Operator

Loss Factor Methodologies Evaluation
Part 1 - Determination of 'Raw' Loss Factors and Load Flow Shift Factors

Final

Teshmont Consultants LP
1190 Waverley Street
Winnipeg, Manitoba
Canada R3T 0P4

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ALBERTA ELECTRIC SYSTEM OPERATOR

LOSS FACTOR METHODOLOGIES EVALUATION
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1 INTRODUCTION

This report discusses the results of full system testing of different methodologies to develop individual generator loss factors to allocate losses to generators for a specific load flow condition.

2 EVALUATION METHODOLOGY

The Alberta Interconnected Electric System (AIES) was used as the basis for all calculations. A set of twelve 2003 load flow conditions as used in AESO's current loss factor calculations was used as the reference power flow cases for all alterative methodologies. The load flow model consists of about 1700 busses, among which 730 have generators, loads or both connected. Bus number 1520 (the 500 kV equivalent of the BC Hydro and WECC system) was designated as the swing bus for the system.

Table 1) presents a summary of the twelve load flow solutions. With the exceptions discussed hereinafter, the summary is based on PSLF Version 13.4 accounting methods. In the load flow data, motor loads are modelled as negative generators; so, total PSLF generation reflects the net component. The contributions of the generation and motor load components have been separated out in the tabulation. The tabulation is similar to the tabulation expected from PSS/E with one exception. PSS/E treats all shunt paths as loads (including transformer no-load losses). PSLF treats transformer shunt paths as magnetizing losses; hence, their contribution to the power balance is included in the 'losses' category.

2.1 Approaches to Loss Factor Calculations

The different methodologies that were evaluated can be categorized into four basic approaches. The four approaches are all based on analyses of loss equations describing the relationship between total transmission system losses and the output of generators connected to the system and modelled in the power flow snapshot of the system operating condition.

2.1.1 Direct Approach

In the direct approach, loss factors are extracted directly from matrix equations describing the relationship between system losses and generation and load at each bus. The equations are examined, and arranged in a form such as the following:

$$
\begin{aligned}
\text{Losses} = {} & K_1(Pg_1, Pg_2, \cdots Pg_n, Pl_1, Pl_2, \cdots Pl_n) \cdot Pg_1 \\
& + K_2(Pg_1, Pg_2, \cdots Pg_n, Pl_1, Pl_2, \cdots Pl_n) \cdot Pg_2 \\
& + \cdots \\
& + K_n(Pg_1, Pg_2, \cdots Pg_n, Pl_1, Pl_2, \cdots Pl_n) \cdot Pg_n \\
& + K_0
\end{aligned}
\qquad \text{Equation (1)}
$$

where:
Pg_i represents the output of generator at bus 'i' and Pl_i represents the magnitude of the load at bus 'i'. In the direct method, the loss factor for generator 'i' is set to the function:

$$
lf_i = K_i(Pg_1, Pg_2, \cdots Pg_n, Pl_1, Pl_2, \cdots Pl_n) \qquad \text{Equation (2)}
$$

This function which when evaluated for each generator is multiplied directly by the generator output, providing an indication of the generators contribution to total system losses. The function can therefore be equated to a loss factor.

The term 'K_0' in Equation (1) represents all components of the total system loss that are independent of generation. This component of the losses is not accounted for during the assignment of losses to generation and therefore will represent the contribution of the direct methodology to the shift factor required to balance the assigned loss equation.

2.1.2 Gradient and Gradient by 2 Methods (Marginal Calculations)

In the gradient method, the loss factor of a single generator is determined from its marginal impact on transmission losses. The gradient, equal to the change in system losses for a given change in individual unit generation can be calculated analytically by differentiation of Equation (1) or numerically using tools such as a load flow to make small changes to individual generator output, and monitoring the impact of the change in system losses. The raw loss factor for each generator is set equal to the gradient. The gradient method may over or under assign losses resulting in a requirement for a shift factor to balance the loss equation.

The gradient method provides a very good estimate of the incremental losses caused by each generator. However, as losses are typically a function of the square of the generation, it does not provide a very good indication of contribution of the total output of the generator to the losses. It can be shown analytically that 100% of the losses can be attributed to both generators and loads based on ½ of their individual gradients. However, as the contributions due to loads must be assigned to the generators, the contribution to losses can be expressed as a shift factor to each of the generator loss factors to balance the loss equation.

2.1.3 Incremental Loss Factor Method

The incremental loss factor method as defined in these investigations is similar to the gradient methods, except the change in output of the generator is set equal to the operating output of the generator. The impact on losses can be determined using differences in load flow losses with and without the generator. Alternatively, an approximate estimate of the incremental loss factors can be determined analytically by solving Equation (1), with and without the generator. The methodology can be used to determine the contribution of losses due to a single generator for a single operating condition, but does not take into account any mutual effects say due to generators in the same location. As a result the methodology may under-(or over-)assign losses and again the shift factor is required to balance the loss equation.

2.1.4 Flow Tracking Method

The flow tracking method is based on tracing the power flows in one direction, upstream from the loads towards the generators or downstream from the generators to the loads. The first approach assigns the losses to the generators, and the latter to the loads. As generators pay for losses in the AESO system, the first approach was investigated. Since exactly 100% of the losses of each branch transmission losses are allocated to one or more of the generators in the system, there, is no over-(or under-)allocation of losses with this methodology. The shift factor is theoretically zero with this method. Several generators in the AIES, however, are not required to pay for losses. The contributions of these generators to total system losses are socialized to other generators using a shift factor.

2.2 Calculation of Gradients

For the gradient methods, several treatments of system load and generation are possible when the gradients are calculated. The treatments that were considered are discussed hereinafter.

2.2.1 Present AESO Swing Bus Method

The present AESO loss factor methodology uses a single swing bus method in which one generator is designated as a swing bus and loss factors are calculated for each other generator based on load flow results. The generator loss factor is equal to the change in losses for a small change in output for the generation for which the loss factor is being calculated. By definition, the raw loss factor of the generator at the swing bus is zero.

2.2.2 Area Load Adjustment

In the area load adjustment method, the generator for which the loss factor is being determined is designated as the swing generator, and load is changed at every bus in the area by a constant ratio. For this calculation the 'area' is selected to be the entire Alberta system. Again loss factor is calculated equal to the change in losses for the resultant change in generation at the swing bus.

2.2.3 Partial Differentiation

A third method for calculating gradient-based loss factors is to set the loss factor for each generator equal to the partial derivative of the loss equation with respect to the output of the generator. This is a purely mathematical expression for loss factor and there is an underlying assumption that all other contributions to the loss equation remain constant.

The loss factor for each generator based on Equation (1) would be equal to the 'direct' loss factor (i.e. the function defined by Equation (2)) plus an additional component equal to the partial derivative of the function with respect to the generator output.

2.3 Solution Methods

In the matrix analysis approaches, loss factors are determined directly from matrices describing the relationship between generator power, bus loads and ac system topology.

The matrix analysis includes an approximate (uncorrected) or exact (corrected) loss matrix describing the dependency of losses on both generation and load. In addition, loss factors were determined using the Kron loss matrix equation in which losses are expressed as only a direct function of generation.

3 METHODOLOGIES EVALUATED

Generator loss factors were determined for each of the methodologies given hereinafter and the results are compared as discussed in Section 4.

3.1 Direct Methodology Using Uncorrected Loss Matrix

In this methodology, the loss factors are determined directly from the coefficients of a system loss matrix.

The system loss matrix is derived from topology and is of the form:

$$\mathbf{R_{uncorr}} = \left(\overline{\mathbf{Y}^{-1}}\right)^{\mathbf{T}} \cdot \mathbf{M}^{\mathbf{T}} \cdot \mathbf{G} \cdot \mathbf{M} \cdot \mathbf{Y}^{-1} \qquad \text{Equation (3)}$$

where:
Y is the nodal admittance matrix for the system
$\left(\overline{\mathbf{Y}^{-1}}\right)^{\mathbf{T}}$ is the transpose of the conjugate of the inverse of the nodal admittance matrix
M is the branch incidence matrix
G is the diagonal matrix of branch conductances.

If '**Y**' is symmetrical, it can be shown that the matrix '**R**' is real, and the uncorrected '**R**' matrix is in effect the real component of the inverse of the nodal admittance matrix **Y**.

Losses can be calculated directly using the expression

$$\text{Losses} = \left(\overline{\mathbf{I}}\right)^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{I} \qquad \text{Equation (4)}$$

where:

I is a vector of current injections corresponding to each generator and load bus of the system and $\left(\overline{\mathbf{I}}\right)^{\mathbf{T}}$ is the transpose of the conjugate of the vector of current injections.

To a first approximation, the loss equation using the system loss matrix can be written in the form:

$$\text{Losses} = (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot (\mathbf{Pg} + \mathbf{Pl}) \qquad \text{Equation (5)}$$

where:
Pg contains the generator output (p.u.) and **Pl** contains the negative values of individual loads (p.u.). Loads are treated as negative generators in this equation.

Since the matrix '**R**' is symmetrical, the equation can be re-written in the form:

$$\text{Losses} = (\mathbf{Pg} + 2\mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{Pg} + \mathbf{Pl}^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{Pl} \qquad \text{Equation (6)}$$

In this expression, losses can be expressed as a function of two components: one component that is independent of generation and another component that is dependent on both load and generation.

The component that is a function of generation is of the form:

$$\text{Losses}_{\text{g}} = \mathbf{LossFactor}^{\mathbf{T}}\mathbf{Pg} \qquad \text{Equation (7)}$$

where:

$$\mathbf{LossFactor}^{\mathbf{T}} = (\mathbf{Pg} + 2\mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \qquad \text{Equation (8)}$$

In this methodology, loss factors were calculated **directly** from the above Equation (6).

The loss matrices '**R**' used in this analysis were the 'uncorrected' matrices, based only on system topology.

In this method, the losses that are a function of only the load component are the major contributor to the unassigned losses. There is an additional component due to errors in loss estimation introduced as a result of using an uncorrected loss matrix.

3.2 Direct Methodology Using Corrected Loss Matrix

If load flow information (such as bus voltages, angles and generator and load power factors) is available, each individual term of the loss matrix can be 'corrected' by the expression:

$$\zeta_{i,j} = \frac{\cos\left((\phi_i - \phi_j) - (\sigma_i - \sigma_j)\right)}{v_i \cdot v_j \cdot \cos(\phi_i) \cdot \cos(\phi_j)}$$ Equation (9)

$$R_{corr_{i,j}} = R_{uncorr_{i,j}} \cdot \zeta_{i,j}$$ Equation (10)

where:
subscripts 'i' and 'j' point to elements of the '**R**' matrix, corresponding to respective buses in the system.

ϕ_i and ϕ_j correspond to the power factor angles at buses 'i' and 'j' respectively.

σ_i and σ_j correspond to the voltage angles at buses 'i' and 'j' respectively.

and v_i and v_j correspond to the magnitudes of the voltages at buses 'i' and 'j' respectively.

With these corrections, Equation (5) above becomes an exact numerical expression of losses.

In this set of calculations the corrected loss matrices were used. Corrections were based on bus voltages, bus angles, and generator and load power factors obtained from the base-case load flow solutions.

With the corrected loss matrix, Equation (5) above gives exactly the same numerical value for total system losses as the load flow.

3.3 Swing Bus Methodology Using Uncorrected Loss Matrix

Equation (5) above can be used to determine the change in losses for a small change in swing bus and loss factor bus generation.

It can be shown that if the loads are unchanged, the change in total system losses due to change in generation is approximately given by:

$$\Delta\text{Losses} = 2(\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{\Delta Pg} \qquad \text{Equation (11)}$$

It is also known that since load is constant, the change in losses is also equal to the sum of the changes in generation, i.e.:

$$\Delta\text{Losses} = \sum_{i} \Delta\text{Pg}_i \qquad \text{Equation (12)}$$

If generation is assumed to be constant at all but the swing bus and the bus at which the loss factor is being calculated, the above equations reduce to two equations with three unknowns: ΔLosses, ΔPg_1 (the change in generation at the bus for which the loss factor is being calculated), and ΔPg_s(the change in generation at the swing bus).

The simultaneous equations can be combined to calculate the ratio:

$$\frac{\Delta\text{Losses}}{\Delta\text{Pg}_1}$$

This is effectively the definition of raw loss factors used in the present AESO loss factor methodology.

For these calculations, the bus 493 (Clover Bar) was used as the swing bus for the calculations. This is consistent with the present AESO swing bus loss factor methodology.

3.4 Swing Bus Methodology Using Corrected Loss Matrix

The calculation discussed in 3.3 above was repeated using the corrected loss matrix. In using the corrected loss matrix for this calculation, the set of assumptions change. For the uncorrected loss matrix calculations, it is mathematically exact to assume that the '**R**' matrix does not change with small changes in load, as the uncorrected '**R**' matrix is a function of only system topology. Assuming the corrected '**R**' matrix to be constant implies that all of the corrections made to the '**R**' matrix are also independent of small changes in generation.

In practice, a small change in generator power output is not likely to significantly alter bus voltages. Load power factors will remain constant, in the same manner as a load flow solution. Generator power factors however are likely to change particularly at the generator where the loss factor is being evaluated and the swing bus. Assuming a constant power factor could lead to undesired consequences.

Any generator operating with a low power factor (for example units connected primarily for var support) would be very susceptible to high loss factor calculations. Assuming the power factor to be constant implies that with every increment in generator output there is a corresponding increase in generator var output. As actual transmission losses are not only a function of MW but also Mvar, the small change in generator output could have a significant impact on total system losses associated with the assumption of a constant '**R**' matrix. The net result is that low power factor generators could be assessed excessively large loss factor penalties or credits.

A second undesirable effect of this assumption is that some generators could be penalized in terms of increased loss factors for supplying vars to the system under conditions when vars are needed on the system. It is also conceivable that some generators and associated loads could receive credits for taking vars from the system under var shortage conditions.

One method of circumventing this issue is to treat all var injections, from both loads and generators as equivalent constant admittance shunt devices. The nodal admittance matrix must be adjusted to include this effect, before the '**R**' matrix is established.

The implication of this treatment of load and generator vars is that the load and generator var injections are treated as being constant. Since bus voltages are assumed to be constant, the vars generated by the equivalent shunt devices are also constant. This is again a reasonable approximation for small changes in generator output.

If the power market evolves to include equivalent var loss factors for both generators and loads, these assumptions would need to be revisited.

3.5 Area Load Methodology Using Uncorrected Loss Matrix

Equation (5) above also can be used to determine the change in losses for a small change in swing bus generation and total system load. If all of the loads in the system are increased by a small percentage (say δ), the total change in system losses can be approximated by the following expression for a given network configuration defined by "**R**" and linearized around a fixed operating condition defined by, "**Pg**" and "**Pl**"

$$\Delta \text{Losses} = 2(\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{\Delta Pg} + \delta \cdot 2(\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{Pl} \qquad \text{Equation (13)}$$

$$\Delta \text{Losses} = \sum_i \Delta \text{Pg}_i + \delta \cdot \sum_i \text{Pl}_i \qquad \text{Equation (14)}$$

If only the generation at the loss factor bus "j" changes, then again the above equations can be reduced to two simultaneous equations with three unknowns (ΔLosses, ΔPg_j, δ).

The simultaneous equations can be combined to again calculate the ratio:

$$\frac{\Delta \text{Losses}}{\Delta \text{Pg}_j} = \frac{2(\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \left(\begin{pmatrix} 0 \\ 1_j \\ 0 \end{pmatrix} - \frac{\mathbf{Pl}}{\sum_i \text{Pl}_i} \right)}{1 - \frac{2(\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{Pl}}{\sum_i \text{Pl}_i}}$$

at the generator bus "j" for which the loss factor is being determined. By linearizing the equations, the above ratio is independent of the magnitude of δ.

For this methodology, the generator for which the loss factor is calculated effectively becomes the swing machine for the system. Hence the loss factors calculated are independent of an arbitrary selection of a swing bus in the system.

3.6 Area Load Methodology Using Corrected Loss Matrix

The calculation method discussed in 3.5 above was repeated using the corrected loss matrix. This method is again subject to the limitations introduced by the assumptions regarding the constant '**R**' matrix discussed in Section 3.4. Generator and load vars are treated as equivalent shunt devices and hence are indirectly assumed to be constant, by the assumption of constant voltages.

As the main function of generator loss factors is to define the relationship between generator power output and transmission losses, it is reasonable to assume that the variation in system load is related only to the active power component, i.e., the change in load vars is zero. The assumption of constant load vars in this corrected '**R**' matrix methodology is therefore reasonable.

3.7 Gradient Methodology Using Uncorrected Loss Matrix

The partial derivative of Equation (5) above with respect to individual generator output can be determined for each generator as follows:

$$\frac{\partial \text{Losses}}{\partial \text{Pg}_i} = 2 \cdot (\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{S(i)} \qquad \text{Equation (15)}$$

where $\mathbf{S(i)}$ is a vector in which the i^{th} element is 1.0 and all other elements are zero.

A vector of all the gradients is simply:

$$\left(\frac{\partial \mathbf{Losses}}{\partial \mathbf{Pg}}\right)^{T} = 2\cdot(\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}}\cdot\mathbf{R} \qquad \text{Equation (16)}$$

The above can be used to allocate losses to generators by multiplying each individual gradient by the generator output.

3.8 Gradient Methodology Using Corrected Loss Matrix

The calculation discussed in 3.7 above can be repeated using the corrected loss matrix. Again the loss factors are dependent on the assumption of a constant '**R**' matrix. This is a mathematically exact assumption, however the impacts of the assumption are the same as discussed in Section 3.4. Load and generator var outputs must be assumed to be constant and be embedded in the '**R**' matrix to avoid unrealistic penalties and credits for vars supplied or absorbed from the system.

3.9 Gradient by 2 Methodology Using Uncorrected Loss Matrix

If Equation (16) above is expanded to include all buses for which generation or load is included, it can be combined with Equation (5) to give:

$$\text{Losses} = \tfrac{1}{2}\left(\frac{\partial \mathbf{Losses}}{\partial \mathbf{Pg}}\right)^{\mathbf{T}}\cdot(\mathbf{Pg}+\mathbf{Pl}) \qquad \text{Equation (17)}$$

I.e. the total losses of the system can be allocated to load and generation buses based on ½ the gradient calculated for each generator and load bus. The component that is due to generation can be determined from:

$$\text{Losses}_{g} = \tfrac{1}{2}\left(\frac{\partial \mathbf{Losses}}{\partial \mathbf{Pg}}\right)^{\mathbf{T}}\cdot\mathbf{Pg} \qquad \text{Equation (18)}$$

and the component of the losses due to load is given by:

$$\text{Losses}_{l} = \tfrac{1}{2}\left(\frac{\partial \mathbf{Losses}}{\partial \mathbf{Pg}}\right)^{\mathbf{T}}\cdot\mathbf{Pl} \qquad \text{Equation (19)}$$

The term $\tfrac{1}{2}\left(\frac{\partial \mathbf{Losses}}{\partial \mathbf{Pg}}\right)$ in Equation (18) can be considered to be a vector of generator raw loss factors and the term 'Losses_{l}' of Equation (19) can be considered to be unassigned losses that are due to loads and which must be factored into the loss balance equation using a shift factor.

One advantage of this methodology is that there is a quantitative explanation of all components of the losses.

3.10 Gradient by 2 Methodology Using Corrected Loss Matrix

The calculation discussed in Section 3.9 above can be repeated using the corrected loss matrix. Again the assumptions regarding the constant '**R**' matrix discussed herein are applicable.

3.11 50% Area Load Methodology Using Uncorrected Loss Matrix

It will be shown that the losses assigned by the area load adjustment methodology are almost twice the actual losses. The loss factors calculated using area load adjustment will be reduced by 50% to determine the average losses and unassigned losses and the shift factor will be recalculated. Please refer to Appendix A

3.12 50% Area Load Methodology Using Corrected Loss Matrix

The loss factors calculated using area load adjustment and the corrected loss matrix will also be reduced by 50% to determine the average losses and unassigned losses and the shift factor will be recalculated. It is shown in Appendix A that loss factors calculated in this manner will account for almost all of the system losses resulting in a shift factor in the order of less than 0.15%. It will be shown numerically in Section 4 that the unassigned losses and resultant shift factor for this methodology are essentially zero.

Again the assumptions regarding the constant '**R**' matrix discussed herein are applicable.

3.13 Kron Loss Matrix Using Direct Methodology

An alternative matrix expression of losses used for optimal power flow solutions is the Kron loss matrix formula.

The equation is of the form:

$$\text{Losses} = \mathbf{Pg}^{\mathbf{T}} \cdot \mathbf{B02} \cdot \mathbf{Pg} + \mathbf{B01}^{\mathbf{T}} \cdot \mathbf{Pg} + \text{B00} \qquad \text{Equation (20)}$$

In the above equation, '$\mathbf{Pg}$' is a vector housing the magnitude of the real output of the generators, $\mathbf{B02}$ is a matrix, $\mathbf{B01}$ is a vector and B00 is a simple scalar.

The loss equation above can be rewritten in the form:

$$\text{Losses} = \left(\mathbf{Pg}^{\mathbf{T}} \cdot \mathbf{B02} + \mathbf{B01}^{\mathbf{T}}\right) \cdot \mathbf{Pg} + \text{B00} \qquad \text{Equation (21)}$$

The bracketed term $\left(\mathbf{Pg}^{\mathbf{T}} \cdot \mathbf{B02} + \mathbf{B01}^{\mathbf{T}}\right)$ can be considered to be a transposed vector of raw loss factors as it allocates all but the component 'B00' of the losses to the generators. The term 'B00' represents an unallocated loss component that will contribute to the shift factor.

3.14 Kron Loss Matrix Using Swing Bus Methodology

The Kron loss equation can be rearranged in a similar fashion to the loss matrix equation to determine loss factors based on the existing swing bus methodology.

$$\Delta\text{Losses} = 2 \cdot \left(\mathbf{Pg}^{\mathbf{T}} \cdot \mathbf{B02} + \mathbf{B01}^{\mathbf{T}}\right) \cdot \Delta\mathbf{Pg} \qquad \text{Equation (22)}$$

It is also known that the change in losses is equal to the sum of the change in losses in all generators, i.e.:

$$\Delta\text{Losses} = \sum_{j} \Delta\text{Pg}_{j} \qquad \text{Equation (23)}$$

If generation is assumed to be constant at all but the swing bus and loss factor bus, the above equations reduce to two equations with three unknowns, namely: ΔLosses, ΔPg_1 (the change in generation at the bus for which the loss factor is being calculated), and ΔPg_s(the change in generation at the swing bus).

The simultaneous equations can be combined to calculate the ratio:

$$\frac{\Delta\text{Losses}}{\Delta\text{Pg}_1}$$

Similar to the corrected loss matrix methods discussed above, this method assumes that the coefficients '**B02**', '**B01**' and 'B00' are constant. While the coefficients are not as straight forward as the loss matrix '**R**' matrix calculations, imbedded in the formulation of the coefficients are corrections for bus voltages, power factors and power angles. As a result, the implications of the assumption of constant coefficients in this methodology are the same as the assumption of constant '**R**' matrix in the corrected loss matrix methodologies.

3.15 Kron Loss Matrix Using Gradient by 2 Method

The partial derivative of Equation (20) above can be determined for each generator as follows:

$$\frac{\partial\text{Losses}}{\partial\text{Pg}_i} = \left(2 \cdot \mathbf{Pg}^{\mathbf{T}} \cdot \mathbf{B02} + \mathbf{B01}^{\mathbf{T}}\right) \cdot \mathbf{S(i)} \qquad \text{Equation (24)}$$

where '**S(i)**' again is a vector in which the i[th] element is 1.0 and all other elements are zero.

The vector

$$\mathbf{G^T} = 2 \cdot \mathbf{Pg^T} \cdot \mathbf{B02} + \mathbf{B01^T} \qquad \text{Equation (25)}$$

therefore contains all of the gradients calculated for each generator.
If the gradient is dominated by the first term in Equation (25), the loss equation can be re-written to:

$$\text{Losses} = \tfrac{1}{2} \cdot \mathbf{G^T} \cdot \mathbf{Pg} + \text{B00} + \varepsilon \qquad \text{Equation (26)}$$

where the term 'ε' represents the error introduced by the approximation by ignoring the B01 component and which must be compensated for in the shift factor along with the B00 term.

Again the coefficients '**B02**', '**B01**' and 'B00' are all assumed to be constant in this methodology. Similar to the loss matrix methodologies discussed in Section 3.8 and 3.10, this is mathematically correct but the implications are the same as discussed in Section 3.14 above.

3.16 Incremental Loss Factor Methodology

The incremental loss factor methodology requires the calculation of the change in losses as the output of a generator is adjusted from no load to its loading as represented in the loadflow. In Equations (13) and (14) above, if the change in generation 'ΔPg_1' is known, the change in losses 'ΔLosses' and the load adjustment factor 'δ' can be determined by solving the two simultaneous equations. The loss factor is set to the numerical value of 'ΔLosses' divided by 'ΔPg_1'. At generator buses where the output is zero, the amount of power reduction is set equal to 0.00001 MW (i.e. the marginal loss factor).

Similar to the rest of the analytical methods discussed above, an assumption is made that the '**R**' loss matrix is unchanged as a result of the change in generation and load.

One of the known limitations of the methodology is its inability to handle TMR situations. Under 'transmission must run' load flow conditions, shut down of a TMR generator (or reduction of power to zero) could result in load flow convergence failure. Although the analytical implementation of the ILF methodology is not subject to convergence issues, the assumptions made would deviate significantly from the practical situation and a significant reduction in accuracy would result.

A second known limitation of this methodology is that generators of different capacity, located at or close to the same bus will be assigned different loss factors. This is not in accordance with the regulations and would require 'special' treatment of nearby generators to meet the regulation requirements. While a mathematical 'special' treatment is possible for these generators, the extent of the definition of 'nearby' will become an issue with this methodology.

3.17 Flow Tracking Methodology

The flow tracking methodology requires that the losses be known for every branch in the system. Each transmission branch loss is assigned to one of the adjacent buses in the direction of tracking; then redistributed among lines proportionally to the flows, again in the direction of tracking. The process is recursive and eventually ends up at the generator buses with the losses assigned to them. Loss factors are then determined based on the assigned losses at each bus.

Several features and known limitations of the methodology are:

1) Losses can only be assigned to either generators or loads, not both at the same time. Any attempt to mix the assignment of losses to both generators and loads would lead to a major change of the algorithm and would require an arbitrary decision on breaking points in the system where the algorithm basis would be switched. As such in the Alberta situation where generators are responsible for losses, any losses associated with exports or DOS loads could not be handled with this algorithm.

2) Loss factor credits are not possible. All loss factors are positive, implying a smaller permitted range of loss factors (after compression) and hence less locational-based generating signals.

3) All load buses are assigned a zero loss factor. This implies that all buses without generation are assigned a loss factor of zero.

4) All generators at buses where load exceeds generation will be assigned a loss factor of zero. This also applies to 'no-loss' areas where load exceeds generation. All generators in such an area would have a loss factor of zero. As a result, the level of encouragement provided for new generation is independent of the magnitude of potential benefit.

5) The methodology fails when situations arise where a generator is small but the var flow in adjacent circuits are large. Such would be the case for units connected primarily for voltage control or for small units connected close to buses with large capacitors. A solution could be to ignore loss factors for small units, but this would require some formula to decide under which situation a unit should be ignored. This will be more complicated than a simple MW criterion as Mvar to MW ratio comes into play.

4 COMPARISON OF METHODOLOGIES

Loss factors were calculated for every generator in the Alberta system for each of the twelve 2003 base-case load flows and for each of the 17 methodologies discussed in Section 3 above. The results of these calculations are summarized herein.

4.1 Required Shift Factor

Table 2) and Table 3) summarize the shift factors associated with each load flow and each methodology. The magnitude of the shift factor is a measure of the ability of each methodology to allocate total system losses on a mathematically defined basis. In this context, shift factor is defined to be the correction that must be made to the loss factor for each individual generator to account for all of the unassigned power (MW) losses in the system. A positive shift factor implies that the methodology would result in an under-assignment of total system losses. I.e., the loss factors of each generator must be increased by the shift factor to recover all of the power flow losses. A negative shift factor implies an over-assignment of losses.

The column "Average Loss Factor" is the ratio of losses to total generation as calculated using a load flow program.

The seasonal average shift factors are simply the average of the shift factors for the three load flows of each season. The annual average shift factor is the average of the four seasonal shift factors (equivalent to the average of the shift factors for all 12 load flows). The average shift factors have no physical interpretation, but are useful for comparing the methodologies.

The shift factors shown in Table 2) and Table 3) are the same. In Table 2), the largest and smallest magnitude shift factors encountered for each methodology, for each power flow, are highlighted. In Table 3) the shift factors for each load flow are compared. The largest and smallest magnitude shift factors encountered for each methodology on a load flow basis are highlighted.

Table 2) indicates that there is no apparent correlation between shift factors and load flow or season. For example, the largest shift factor does not always occur for a specific season or load flow condition, independent of methodology. For some methodologies the largest shift factor occurs under winter peak conditions but for others the smallest shift factor occurs for that load flow condition.

Table 3) however does start to indicate a trend in results. The 50% area load adjustment methodologies (both corrected and uncorrected matrices) account for all of the smallest shift factors calculated. The largest shift factors occur with the following methodologies:

- uncorrected R matrix, area load adjustment
- corrected R matrix, Direct methodology
- Kron Matrix, Swing bus methodology

The corrected and uncorrected loss matrix swing bus methodologies require similar shift factors. Both under-allocate losses, and both require shift factors similar in magnitude to the current AESO swing bus methodology.

The corrected and uncorrected loss matrix area load adjustments again require similar shift factors. Both over-allocate losses. In fact, both methods over-allocate by an amount that is almost equal to the average loss factor, particularly for the corrected loss matrix methodology.

If the loss factors computed with this method are reduced by a factor of 2, resulting in loss factors that are 50% of the area load adjustment methodology, the required shift factor as indicated above is extremely small.

The shift factors required for the uncorrected and corrected loss matrix direct methodologies are not similar in magnitude. This indicates that the methodology is extremely sensitive to assumptions made in the creation of the loss matrix. Both approaches under-assign losses but the shift factors required for the corrected matrix methodology are actually greater than the average system loss factor implying that the total losses accounted for by the methodology are negative.

Similar to the load area adjustment methodology, the loss matrix gradient method significantly over-assigns losses. The corresponding methodology with ½ gradients under-assigns losses, but in this case, the shift factor calculated using the corrected loss matrix is actually greater that the shift factor calculated using an uncorrected matrix. In the corrected matrix, the shift factor is due entirely to the contribution of the system loads to the losses. In the uncorrected method, inaccuracies introduced by the uncorrected loss matrix tend to counteract the effects of the loads. This would not occur if system voltage profiles were lower.

The direct and gradient by 2 Kron matrix based methodologies slightly under-assign losses with the gradient by 2 methodology requiring the lowest shift factor. The Kron matrix swing bus methodology shows less consistent results between load flows.

4.2 Range of Loss Factors

The Alberta Department of Energy has indicated that all 'normalized' loss factors must be no greater than twice the average system loss factor and no less than the negative value of the average system loss factor

The range of loss factors after application of the shift factors described in Section 4.1 provides an indication of the extent that loss factors calculated using each methodology would exceed the Department's requirements. Table 4) summarizes the variations in loss factors that could be expected and provides an indication of the degree of ultimate loss factor correction that eventually would have to be applied.

For methodologies (such as the incremental loss factor methodology) with a relatively large range in loss factors (37%) and a large number of generators exceeding the criteria (78), the compression algorithm proposed in Part 3 of the report would not be satisfactory. The majority of the units would be either at maximum charge or maximum credit and the locational-based incentives required by the board would be lost. To achieve a distribution of loss factors closer to the intent of compression, other algorithms such as the linear compression algorithm would be required, reducing the locational-based signals of those generators originally within the limits.

In Table (4), the "maximum loss factor" is the largest adjusted seasonal loss factor (12 case average) based on individual generators. "Minimum loss factor" is the smallest (or largest negative) value and "range of loss factors" is the difference between the two extremes.

The table also indicates the number of generators with loss factors greater than the criteria and the number of generators with loss factors less than the criteria along with the total. Although the loss factors on which the table is based have been adjusted to take into account and balance all of the power flow losses, an additional correction would be required to take into account differences between load flow losses and forecast generator volumes and losses. The next level of correction will shift the range and as a result, the number of generators with loss factors greater than the maximum permitted may change (say increase), but the number of generators with loss factors less than criteria will also change (i.e. decrease) but the change in total number of generators exceeding the criteria should not be significant.

The Kron matrix direct methodology has the lowest range of loss factors and as a result also has the least number of loss factors that exceed the criteria. The uncorrected loss matrix swing-bus methodology has the largest range and consequently the largest number of generators (86) exceeding the criteria.

4.3 Seasonal Volatility

The Alberta Department of Energy has also indicated that each generator will be assigned a single loss factor. This loss factor will represent the contribution of the generator to losses on an annual basis (at minimum). As the loss factors will be based on some average (weighted or un-weighted) of loss factors calculated using load flows as a starting point, the seasonal volatility of the loss factor becomes an indicator of the degree of accuracy that can be expected when assigning energy based loss factors.

Table 4) also indicates the seasonal volatility of loss factors for each methodology. Volatility is expressed as the largest range in individual generator loss factors over each of the four seasons.

Loss factors calculated using the Kron matrix direct and gradient by 2 methodologies are least sensitive to the variations introduced by the four seasons. This is followed closely by the 50% area load adjustment methodologies and the loss matrix gradient by 2 methods. The range in seasonal volatility for these six methods is from 4.01 to 5.1%.

The flow-tracking methodology has by far the poorest (largest) seasonal volatility at 78.73%. The 'R' loss matrix swing bus methodologies are next with seasonal volatilities of 11.45% and 11.37% for the uncorrected and corrected 'R'-matrix methodologies respectively.

4.4 Ranking of Alternative Methodologies.

Each of the methodologies has certain advantages and disadvantages. To quantify the overall assessment of the methodologies, a ranking has been determined for each of the metrics.

The first metric assessed was the load flow adjustment shift factor. Table 3) indicated that the magnitude of shift factor was dependent on not only the methodology but also the individual load flow condition and the season. To assess this metric, the methodologies were ranked for each load flow condition from 1 to 17, depending on the magnitude of load flow shift factor as shown in Table 5). The methodologies were also ranked in terms of the seasonal and annual loss factors from 1 to 17. A ranking of 1 indicates the most desirable while a ranking of 17 is least desirable.

A weighted average of each of the individual rankings was determined for each methodology. The weightings assigned were:

Individual load flows	1/36
Individual Seasons	3/36
Annual Shift Factor	12/36

The weightings effectively give equal weight (1/3) to all of the load flows, all of the seasons and the annual shift factor.

Table 5) indicates that the methodology with the lowest ranking or minimum overall shift factor is the corrected loss matrix, 50% area load adjustment methodology. The methodology with the highest weighted average is the loss matrix direct methodology. The methodologies have been ranked from 1 to 17 based on the weighted average of the individual ranking as shown in the table.

The methodologies have also been ranked from 1 to 17 based on each of the other metrics discussed above. These are:

- The number of generators that exceed the loss factor limits
- The range of loss factors
- Seasonal Volatility

A fifth metric also considered was the dependency of the methodology on selection of swing bus. A problem associated with those methodologies that are dependent on the selection of swing bus for the system is actually designating the appropriate swing bus. The most appropriate swing bus may need to change with changes in topology and system loading conditions. Those methodologies with no dependence on swing bus selection were assigned a rank of 1 (all tied for 1st place). Those methodologies where there is a dependence on swing bus selection were assigned a ranking of 17 (tied for last place).

Each of the metric rankings were assigned an equal weighting and a weighted sum factoring all of the metrics was determined. The methodology for ranking of alternatives is shown in Table 6). The ranking is based on the weighted sum of the individual rankings.

Based on this assessment method, both loss matrix 50% area load adjustment methodologies rank in the top two, with the corrected loss matrix method on top followed by the uncorrected loss matrix method.

The best (lowest) ranking of methodologies based on the Kron matrix formula is the direct methodology in overall position number 5. The flow-tracking approach ranked 7th while the ranking of the incremental loss factor methodology was 10th overall. The loss matrix swing bus methodologies (close to the current methodology) are ranked last.

As the methodologies can be separated into two distinct groups, namely those based on corrected matrices and those based on uncorrected matrices, the ranking process described above was repeated for each group. The comparable rankings are shown in Table 7) and Table 8).

The 50% area load adjustment methodology remains at the top in both categories. The Kron loss formula based methods improve to positions 2 and 4 in the corrected matrix grouping with the direct methodology in position 2. The swing bus methodology remains in last place in both groupings.

4.5 Qualitative Assessments

4.5.1 Change in Relative Order of Loss Factors

A qualitative assessment was made on selected methodologies relating to the possible degree of acceptability of the methodology to stakeholders. **Figure 1**) compares the relative position in terms of loss factors of the average loss factor for each of the load flow areas. The order of the area was selected based on the highest to lowest average loss factors for the 50% area load adjustment methodology. The plot designated 'Rmat_Ald50' (50% area load adjustment methodology) is therefore a straight line. With the exception of load flow areas 55 and 17 where the ranking of the average loss factors is swapped, the ranking of the 50% area load adjustment methodology is the same as the ranking of the 'RMat_Swg_Cor' (corrected 'R' loss matrix, swing bus methodology). The latter methodology is similar to the current methodology. While the magnitudes of the loss factors will be different between the two methodologies, there will be few 'winners' or 'losers' in terms of relative order of the loss factors.

The loss factors for the 'RMat_Der2_Cor' (corrected 'R' loss matrix 50% gradient methodology) are ranked in the same order as the 50% area load adjustment methodology. Loss factors for 'ILF' (incremental loss factor methodology) follow a similar overall trend to the current methodology, however there are more differences in ranking and as a result more 'winners' and 'losers' in terms of relative competitiveness from loss charge considerations.

The flow tracking methodology would result in a significant shift in the distribution of charges and credits. Generators in areas that presently have similar loss factors could end up with vastly different loss factors ranging in some areas from maximum credits to maximum charges.

While the objective of the methodology investigation was not to maintain a 'status quo', all other factors being equal, 50% area load adjustment and 50% gradient methodologies, would be preferred to the other methods compared.

4.5.2 Transmission Must Run Capability

As the incremental loss factor methodology cannot accurately handle TMR generating conditions, a 'special' treatment of these generators would be required. The regulations require that all generators at the same bus have the same loss factor. This is not an inherent feature of the incremental loss factor methodology and again special treatment of generators within the same area may be required to achieve this objective. While a requirement for 'special' treatment has not been factored into the quantitative assessment given above, all other factors being equal, methodologies where 'special' treatment is not required would be preferable to the incremental loss factor methodology.

4.5.3 Var Flow Issues

Complications associated with large generator var flow have been taken care of in the 'R' loss matrix and Kron Matrix methodologies by converting all var load and generator var outputs to equivalent shunt devices before determining the loss factors.

As indicated above, the flow tracking methodology would require 'special' treatment for small generators connected to branches with large var flows. Similar to the incremental loss factor methodology above, all other factors being equal, methodologies where 'special' treatment is not required would be preferable to the flow tracking methodology.

4.5.4 Locational Based Signals

The quantitative evaluation of alternative methodologies took into account loss factors outside of the permitted loss factor range, but did not factor in the loss of locational-based signals for methodologies where the loss factor range is less than the permitted range. With the exception of the flow tracking methodology loss factor compression will force all generator loss factors to be within the permitted loss factor range, and the range is such that the locational-based generating signal is maintained.

With the flow tracking methodology, as there are no credits in the raw loss factors, compression will apply only to charges and the strength of the resultant locational-based signal will be about 2/3 of the strength of the signals created by the other methodologies. All other factors being equal, methodologies where the locational-based signal is stronger would be preferable to the flow tracking methodology.

5 RECOMMENDATION

Based on the rankings of alternatives, it is clear that the loss matrix 50% area load adjustment methodology is the best approach to allocating losses to generators. The methodology results in a small load flow shift factor. Generator loss factors are independent of the selection of the swing bus for the system. I.e. when the loss factor is calculated for each generator, the bus to which the generator is connected must become the swing bus for the system. The number of generators that are likely to drive loss factor compression is small (in the order of 12) and the extent of compression required is low with a requirement to reduce the loss factor range from about 18.5% to three times the average loss factor or about 15%.

One of the other requirements of the Alberta Department of Energy is that with the chosen methodology, loss factors of nearby (electrical) generators be similar.

Loss factors were calculated for each generator in each of the load flow areas. The results are given in Table 9) for the corrected loss matrix and Table 10) for the uncorrected loss matrix, 50% area load adjustment methodologies. In Table 9), the variation in adjusted loss factors varies from as low as 0.05% in load flow area 43 (Sheerness) to as high as 7.03% in load flow area 97 (designated as "IPP site"). The variation in area 40 (Lake Wabamun accounting for the majority of the Alberta system generation) is only 0.76%.

Although there is a slight shift in the loss factors within each area when calculated with the uncorrected loss matrix, the range remains about the same, in particular in area 40 where the range of loss factor variation remains low at 1.31%.

A comparison of the average loss factors for each of the load flow areas and for both the corrected and un-corrected loss matrices is given in Figure 2). The pattern evident in the average loss factors for each load flow area for the uncorrected matrix methodology is similar to the corresponding pattern with the corrected matrix methodology. However, the loss factors (both penalties and credits) are sufficiently different so as to limit the usefulness of the uncorrected matrix methodology.

The uncorrected matrix methodology has advantages in terms of transparency. These methodologies eliminate the variation introduced into the loss factor calculation as a result of load flow solution.

For the existing methodology, loss factors for all new generators are based on information deemed to be confidential by the generators. This information is embedded in the load flows and as a result, the load flows themselves have also been deemed to be confidential. If the loss factor calculations were based on an uncorrected loss matrix, the calculation would be dependent only on system topology and assumed distribution of generation and loads. System topology and data is openly available through TASMO. The distribution of loads is not considered confidential and the stacking order for generation is public information. The only unavailable quantity would be the amount of generation assumed for each entry in the stacking order as this information was considered to be confidential. It should be possible, however, to establish a reasonable estimate of the generation distribution based on historical system performance and posted representative system load flows.

If an approach to loss factor calculations is adopted that is based on historical utilization of the transmission system by each generator, the confidentiality issue may disappear, and all aspects of the loss factor calculations could become public.

In this case, the value of the uncorrected matrix methodologies diminishes. The corrected matrix methodology should be adopted because of its more accurate distribution of load flow losses.

The recommended methodology therefore for determining load flow based 'raw' loss factors for generators is the corrected loss matrix, 50 % area load adjustment methodology.

Table 1 Load Flow Solution Summary

	WnPk	WnMd	WnLw	SpPk	SpMd	SpLw	SmPk	SmMd	SmLw	FlPk	FlMd	FlLw
Total Generation	**8456.8**	**7845.7**	**7548.6**	**7978.7**	**7554.2**	**7297.3**	**8269.2**	**7594.3**	**7331.2**	**8390.3**	**7737.9**	**7459.0**
Generation	8423.8	7812.7	7515.6	7945.7	7521.2	7264.3	8236.2	7561.3	7298.2	8357.3	7704.9	7426.0
Negative loads	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0
Total Imports	**258.9**	**-8.3**	**-604.1**	**98.0**	**-70.3**	**-673.3**	**94.6**	**-33.1**	**-706.8**	**433.4**	**3.6**	**-619.1**
SPC Imports	100.0	-0.1	-75.0	100.0	0.0	-75.0	100.0	-0.1	-75.0	100.0	0.0	-75.0
BC Imports	158.9	-8.2	-529.1	-2.0	-70.3	-598.3	-5.4	-33.0	-631.8	333.4	3.6	-544.1
Total Loads	**8345.3**	**7473.8**	**6536.0**	**7718.0**	**7144.5**	**6236.6**	**8020.0**	**7229.3**	**6236.2**	**8468.4**	**7393.4**	**6449.7**
Constant P Loads	8043.6	7172.4	6234.4	7453.2	6879.6	5971.6	7725.8	6935.1	5942.0	8173.4	7098.4	6154.7
Motor Loads	276.2	276.2	276.2	239.4	239.4	239.4	294.2	294.2	294.2	295.0	295.0	295.0
Shunts	25.5	25.2	25.3	25.4	25.4	25.6	0.0	0.0	0.0	0.0	0.0	0.0
Load Flow Losses	**370.5**	**363.5**	**408.5**	**358.7**	**339.5**	**387.4**	**343.8**	**331.9**	**388.2**	**355.3**	**348.1**	**390.2**
Generation + imports less loads	370.5	363.5	408.5	358.7	339.5	387.4	343.8	331.9	388.2	355.3	348.1	390.2
Mismatch	**0.000**	**0.001**	**0.039**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**

Table 2 Load Flow Shift Factors Required For Each Methodology (Part "a")

		Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix
Loading Condition	Average Loss Factor	Swing Bus Methodology	Swing Bus Methodology	Area Load Methodology	Area Load Methodology	50% Area Load Methodology	50% Area Load Methodology	Direct Methodology	Direct Methodology	Gradient Methodology	Gradient Methodology
WnPk	4.77%	2.07%	1.43%	-4.79%	-4.57%	-0.01%	0.10%	2.07%	7.87%	-2.84%	-0.70%
WnMd	5.16%	3.75%	2.88%	-5.25%	-4.99%	-0.04%	0.09%	1.73%	7.47%	-3.62%	-1.84%
WnLw	6.42%	4.19%	3.99%	-7.82%	-6.37%	-0.70%	0.02%	0.86%	6.58%	-6.77%	-5.72%
SpPk	5.01%	2.06%	1.48%	-4.93%	-4.84%	0.04%	0.09%	1.91%	7.87%	-3.19%	-1.67%
SpMd	5.05%	3.30%	2.43%	-5.09%	-4.90%	-0.02%	0.07%	1.63%	8.21%	-3.66%	-2.16%
SpLw	6.41%	3.38%	3.37%	-7.66%	-6.47%	-0.62%	-0.03%	0.93%	6.85%	-6.87%	-7.16%
SmPk	4.32%	1.69%	1.20%	-4.80%	-4.15%	-0.24%	0.08%	1.79%	6.67%	-2.95%	-0.43%
SmMd	4.55%	3.44%	2.67%	-5.12%	-4.42%	-0.29%	0.06%	1.34%	6.43%	-3.66%	-1.85%
SmLw	6.03%	3.04%	3.43%	-8.02%	-6.05%	-0.99%	-0.01%	0.57%	5.93%	-7.14%	-6.41%
FlPk	4.22%	1.03%	0.58%	-4.50%	-4.06%	-0.14%	0.08%	0.57%	6.26%	-2.61%	-0.51%
FlMd	4.65%	3.70%	2.93%	-5.36%	-4.53%	-0.35%	0.06%	1.30%	5.64%	-3.81%	-2.09%
FlLw	5.86%	3.24%	3.42%	-7.70%	-5.86%	-0.92%	0.00%	0.74%	5.55%	-6.77%	-5.73%
Winter Average		3.34%	2.77%	-5.95%	-5.31%	-0.25%	0.07%	1.55%	7.31%	-4.41%	-2.75%
Spring Average		2.91%	2.43%	-5.90%	-5.40%	-0.20%	0.04%	1.49%	7.64%	-4.57%	-3.66%
Summer Average		2.72%	2.43%	-5.98%	-4.88%	-0.51%	0.04%	1.23%	6.34%	-4.58%	-2.90%
Fall Average		2.66%	2.31%	-5.85%	-4.82%	-0.47%	0.05%	0.87%	5.82%	-4.40%	-2.78%
Annual Average		2.91%	2.48%	-5.92%	-5.10%	-0.36%	0.05%	1.29%	6.78%	-4.49%	-3.02%

		Uncorrected R-matrix	Corrected R-matrix	Kron Matrix	Kron Matrix	Kron Matrix	Corrected R-matrix	Branch Loss Matrix
Loading Condition	Average Loss Factor	Gradient/2 Methodology	Gradient/2 Methodology	Direct Methodology	Swing Bus Methodology	Gradient/2 Methodology	ILF Methodology	Flow Tracking
WnPk	4.77%	0.96%	2.03%	1.19%	-11.95%	0.65%	-2.50%	0.51%
WnMd	5.16%	0.77%	1.66%	1.28%	-7.63%	0.68%	-2.89%	0.50%
WnLw	6.42%	-0.18%	0.35%	1.82%	2.50%	0.40%	-3.68%	0.55%
SpPk	5.01%	0.91%	1.67%	1.26%	-9.02%	0.70%	-2.68%	0.56%
SpMd	5.05%	0.69%	1.45%	1.27%	-5.68%	0.75%	-2.83%	0.50%
SpLw	6.41%	-0.23%	-0.37%	1.98%	9.49%	0.38%	-3.57%	0.54%
SmPk	4.32%	0.68%	1.94%	0.91%	-11.19%	0.49%	-2.13%	0.20%
SmMd	4.55%	0.44%	1.35%	0.90%	-5.28%	0.47%	-2.45%	0.18%
SmLw	6.03%	-0.55%	-0.19%	1.72%	5.81%	0.06%	-3.08%	0.19%
FlPk	4.22%	0.81%	1.86%	0.86%	-12.00%	0.46%	-1.93%	0.20%
FlMd	4.65%	0.42%	1.28%	0.89%	-5.52%	0.39%	-2.51%	0.17%
FlLw	5.86%	-0.45%	0.07%	1.47%	3.33%	0.11%	-3.18%	0.18%
Winter Average		0.52%	1.35%	1.43%	-5.69%	0.58%	-3.02%	0.52%
Spring Average		0.46%	0.92%	1.50%	-1.74%	0.61%	-3.03%	0.53%
Summer Average		0.19%	1.03%	1.18%	-3.55%	0.34%	-2.55%	0.19%
Fall Average		0.26%	1.07%	1.07%	-4.73%	0.32%	-2.54%	0.18%
Annual Average		0.36%	1.09%	1.29%	-3.93%	0.46%	-2.79%	0.36%

Legend
Largest Shift Factor per Methodology
Smallest Shift Factor per Methodology

Table 3 Load Flow Shift Factors Required For Each Methodology (Part "b")

Loading Condition	Average Loss Factor	Uncorrected R-matrix Swing Bus Methodology	Corrected R-matrix Swing Bus Methodology	Uncorrected R-matrix Area Load Methodology	Corrected R-matrix Area Load Methodology	Uncorrected R-matrix 50% Area Load Methodology	Corrected R-matrix 50% Area Load Methodology	Uncorrected R-matrix Direct Methodology	Corrected R-matrix Direct Methodology	Uncorrected R-matrix Gradient Methodology	Corrected R-matrix Gradient Methodology
WnPk	4.77%	2.07%	1.43%	-4.79%	-4.57%	-0.01%	0.10%	2.07%	7.87%	-2.84%	-0.70%
WnMd	5.16%	3.75%	2.88%	-5.25%	-4.99%	-0.04%	0.09%	1.73%	7.47%	-3.62%	-1.84%
WnLw	6.42%	4.19%	3.99%	-7.82%	-6.37%	-0.70%	0.02%	0.86%	6.58%	-6.77%	-5.72%
SpPk	5.01%	2.06%	1.48%	-4.93%	-4.84%	0.04%	0.09%	1.91%	7.87%	-3.19%	-1.67%
SpMd	5.05%	3.30%	2.43%	-5.09%	-4.90%	-0.02%	0.07%	1.63%	8.21%	-3.66%	-2.16%
SpLw	6.41%	3.38%	3.37%	-7.66%	-6.47%	-0.62%	-0.03%	0.93%	6.85%	-6.87%	-7.16%
SmPk	4.32%	1.69%	1.20%	-4.80%	-4.15%	-0.24%	0.08%	1.79%	6.67%	-2.95%	-0.43%
SmMd	4.55%	3.44%	2.67%	-5.12%	-4.42%	-0.29%	0.06%	1.34%	6.43%	-3.66%	-1.85%
SmLw	6.03%	3.04%	3.43%	-8.02%	-6.05%	-0.99%	-0.01%	0.57%	5.93%	-7.14%	-6.41%
FlPk	4.22%	1.03%	0.58%	-4.50%	-4.06%	-0.14%	0.08%	0.57%	6.26%	-2.61%	-0.51%
FlMd	4.65%	3.70%	2.93%	-5.36%	-4.53%	-0.35%	0.06%	1.30%	5.64%	-3.81%	-2.09%
FlLw	5.86%	3.24%	3.42%	-7.70%	-5.86%	-0.92%	0.00%	0.74%	5.55%	-6.77%	-5.73%
Winter Average		3.34%	2.77%	-5.95%	-5.31%	-0.25%	0.07%	1.55%	7.31%	-4.41%	-2.75%
Spring Average		2.91%	2.43%	-5.90%	-5.40%	-0.20%	0.04%	1.49%	7.64%	-4.57%	-3.66%
Summer Average		2.72%	2.43%	-5.98%	-4.88%	-0.51%	0.04%	1.23%	6.34%	-4.58%	-2.90%
Fall Average		2.66%	2.31%	-5.85%	-4.82%	-0.47%	0.05%	0.87%	5.82%	-4.40%	-2.78%
Annual Average		2.91%	2.48%	-5.92%	-5.10%	-0.36%	0.05%	1.29%	6.78%	-4.49%	-3.02%

Loading Condition	Average Loss Factor	Uncorrected R-matrix Gradient/2 Methodology	Corrected R-matrix Gradient/2 Methodology	Kron Matrix Direct Methodology	Kron Matrix Swing Bus Methodology	Kron Matrix Gradient/2 Methodology	Corrected R-matrix ILF Methodology	Branch Loss Matrix Flow Tracking
WnPk	4.77%	0.96%	2.03%	1.19%	-11.95%	0.65%	-2.50%	0.51%
WnMd	5.16%	0.77%	1.66%	1.28%	-7.63%	0.68%	-2.89%	0.50%
WnLw	6.42%	-0.18%	0.35%	1.82%	2.50%	0.40%	-3.68%	0.55%
SpPk	5.01%	0.91%	1.67%	1.26%	-9.02%	0.70%	-2.68%	0.56%
SpMd	5.05%	0.69%	1.45%	1.27%	-5.68%	0.75%	-2.83%	0.50%
SpLw	6.41%	-0.23%	-0.37%	1.98%	9.49%	0.38%	-3.57%	0.54%
SmPk	4.32%	0.68%	1.94%	0.91%	-11.19%	0.49%	-2.13%	0.20%
SmMd	4.55%	0.44%	1.35%	0.90%	-5.28%	0.47%	-2.45%	0.18%
SmLw	6.03%	-0.55%	-0.19%	1.72%	5.81%	0.06%	-3.08%	0.19%
FlPk	4.22%	0.81%	1.86%	0.86%	-12.00%	0.46%	-1.93%	0.20%
FlMd	4.65%	0.42%	1.28%	0.89%	-5.52%	0.39%	-2.51%	0.17%
FlLw	5.86%	-0.45%	0.07%	1.47%	3.33%	0.11%	-3.18%	0.18%
Winter Average		0.52%	1.35%	1.43%	-5.69%	0.58%	-3.02%	0.52%
Spring Average		0.46%	0.92%	1.50%	-1.74%	0.61%	-3.03%	0.53%
Summer Average		0.19%	1.03%	1.18%	-3.55%	0.34%	-2.55%	0.19%
Fall Average		0.26%	1.07%	1.07%	-4.73%	0.32%	-2.54%	0.18%
Annual Average		0.36%	1.09%	1.29%	-3.93%	0.46%	-2.79%	0.36%

Legend
- Largest Shift Factor Per Load Flow or Season
- Smallest Shift Factor Per Load Flow or Season

Table 4 Range of Loss Factors per Methodology

	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix
	Swing Bus Methodology	Swing Bus Methodology	Area Load Methodology	Area Load Methodology	50% Area Load Methodology	50% Area Load Methodology
Maximum Loss Factor	28.72%	18.88%	26.57%	17.82%	15.89%	11.51%
Minimum Loss Factor	-33.14%	-21.29%	-29.76%	-19.21%	-12.28%	-7.00%
Range of Loss Factors	61.86%	40.17%	56.33%	37.03%	28.17%	18.52%
No. Greater Than Maximum Permitted	20	20	20	20	19	3
No. Less Than Minimum Permitted	66	60	63	58	38	9
No of Generators Exceeding Criteria	86	80	83	78	57	12
Seasonal Volatility	11.45%	11.37%	10.22%	10.31%	4.87%	4.92%

	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix
	Direct Methodology	Direct Methodology	Gradient Methodology	Gradient Methodology	Gradient/2 Methodology	Gradient/2 Methodology
Maximum Loss Factor	16.15%	7.95%	26.91%	18.12%	16.06%	11.66%
Minimum Loss Factor	-18.13%	-24.16%	-30.34%	-19.77%	-12.57%	-7.28%
Range of Loss Factors	34.28%	32.12%	57.25%	37.89%	28.62%	18.95%
No. Greater Than Maximum Permitted	17	0	20	20	20	3
No. Less Than Minimum Permitted	41	19	64	60	40	9
No of Generators Exceeding Criteria	58	19	84	80	60	12
Seasonal Volatility	8.07%	6.78%	10.43%	10.69%	4.98%	5.10%

	Kron Matrix	Kron Matrix	Kron Matrix	Corrected R-matrix	Branch Loss Matrix
	Direct Methodology	Swing Bus Methodology	Gradient/2 Methodology	ILF Methodology	Flow Tracking
Maximum Loss Factor	10.33%	17.29%	11.23%	16.36%	31.93%
Minimum Loss Factor	-5.30%	-18.06%	-6.35%	-20.74%	0.36%
Range of Loss Factors	15.62%	35.35%	17.57%	37.11%	31.57%
No. Greater Than Maximum Permitted	0	20	3	20	4
No. Less Than Minimum Permitted	1	57	2	58	0
No of Generators Exceeding Criteria	1	77	5	78	4
Seasonal Volatility	4.01%	9.02%	4.46%	7.28%	78.73%

Legend
- Largest Magnitude per Methodology (red)
- Smallest Magnitude per Methodology (green)

Table 5 Ranking of Methodologies Based on Magnitude of Shift Factor

Loading Condition	Average Loss Factor	Uncorrected R-matrix Swing Bus Methodology	Corrected R-matrix Swing Bus Methodology	Uncorrected R-matrix Area Load Methodology	Corrected R-matrix Area Load Methodology	Uncorrected R-matrix 50% Area Load Methodology	Corrected R-matrix 50% Area Load Methodology	Uncorrected R-matrix Direct Methodology	Corrected R-matrix Direct Methodology	Uncorrected R-matrix Gradient Methodology	Corrected R-matrix Gradient Methodology
WnPk	4.77%	11	8	15	14	1	2	10	16	13	5
WnMd	5.16%	13	10	15	14	1	2	8	16	12	9
WnLw	6.42%	12	11	17	14	6	1	7	15	16	13
SpPk	5.01%	11	7	15	14	1	2	10	16	13	8
SpMd	5.05%	12	10	15	14	1	2	8	17	13	9
SpLw	6.41%	10	9	16	12	6	1	7	13	14	15
SmPk	4.32%	9	8	15	14	3	1	10	16	13	4
SmMd	4.55%	12	11	15	14	3	1	7	17	13	9
SmLw	6.03%	9	11	17	14	7	1	6	13	16	15
FlPk	4.22%	10	7	15	14	2	1	6	16	13	5
FlMd	4.65%	12	11	15	14	3	1	8	17	13	9
FlLw	5.86%	10	12	17	15	7	1	6	13	16	14
Winter Average		12	10	16	14	2	1	8	17	13	9
Spring Average		11	10	16	15	2	1	7	17	14	13
Summer Average		11	9	16	15	5	1	8	17	14	12
Fall Average		11	9	17	15	5	1	6	16	13	12
Annual Average		11	9	16	15	4	1	7	17	14	12
Weighted Average		11.06	9.36	15.94	14.56	3.64	1.11	7.33	16.39	13.75	11.03
Overall Ranking		12	9	16	15	4	1	7	17	14	11

Loading Condition	Average Loss Factor	Uncorrected R-matrix Gradient/2 Methodology	Corrected R-matrix Gradient/2 Methodology	Kron Matrix Direct Methodology	Kron Matrix Swing Bus Methodology	Kron Matrix Gradient/2 Methodology	Corrected R-matrix ILF Methodology	Branch Loss Matrix Flow Tracking
WnPk	4.77%	6	9	7	17	4	12	3
WnMd	5.16%	5	7	6	17	4	11	3
WnLw	6.42%	2	3	8	9	4	10	5
SpPk	5.01%	5	9	6	17	4	12	3
SpMd	5.05%	4	7	6	16	5	11	3
SpLw	6.41%	2	3	8	17	4	11	5
SmPk	4.32%	6	11	7	17	5	12	2
SmMd	4.55%	4	8	6	16	5	10	2
SmLw	6.03%	5	4	8	12	2	10	3
FlPk	4.22%	8	11	9	17	4	12	3
FlMd	4.65%	5	7	6	16	4	10	2
FlLw	5.86%	5	2	8	11	3	9	4
Winter Average		3	6	7	15	5	11	4
Spring Average		3	6	8	9	5	12	4
Summer Average		3	6	7	13	4	10	2
Fall Average		3	7	8	14	4	10	2
Annual Average		2	6	8	13	5	10	3
Weighted Average		3.25	6.33	7.53	13.64	4.50	10.53	3.06
Overall Ranking		3	6	8	13	5	10	1

Legend
- Largest Ranking Per Load Flow or Season
- Smallest Ranking Per Load Flow or Season

Table 6 Overall Ranking Of Methodologies

Criteria	Weighting	Uncorrected R-matrix Swing Bus Methodology	Corrected R-matrix Swing Bus Methodology	Uncorrected R-matrix Area Load Methodology	Corrected R-matrix Area Load Methodology	Uncorrected R-matrix 50% Area Load Methodology	Corrected R-matrix 50% Area Load Methodology
Shift Factor	1	12	9	16	15	4	1
Number of Generators That Exceed the Limits	1	17	13	15	11	7	4
Range of Loss Factors	1	17	14	15	11	5	3
Seasonal Volatility	1	16	15	11	12	3	4
Swing Independent	1	15	15	1	1	1	1
Weighted Sum		15.40	13.20	11.60	10.00	4.00	2.60
Final Ranking		17	16	13	11	2	1

Criteria	Weighting	Uncorrected R-matrix Direct Methodology	Corrected R-matrix Direct Methodology	Uncorrected R-matrix Gradient Methodology	Corrected R-matrix Gradient Methodology	Uncorrected R-matrix Gradient/2 Methodology	Corrected R-matrix Gradient/2 Methodology
Shift Factor	1	7	17	14	11	3	6
Number of Generators That Exceed the Limits	1	8	6	16	13	9	4
Range of Loss Factors	1	9	8	16	13	6	4
Seasonal Volatility	1	9	7	13	14	5	6
Swing Independent	1	1	1	1	1	1	1
Weighted Sum		6.80	7.80	12.00	10.40	4.80	4.20
Final Ranking		8	9	15	12	4	3

Criteria	Weighting	Kron Matrix Direct Methodology	Kron Matrix Swing Bus Methodology	Kron Matrix Gradient/2 Methodology	Corrected R-matrix ILF Methodology	Branch Loss Matrix Flow Tracking
Shift Factor	1	8	13	5	10	1
Number of Generators That Exceed the Limits	1	1	10	3	11	2
Range of Loss Factors	1	1	10	2	12	7
Seasonal Volatility	1	1	10	2	8	17
Swing Independent	1	15	15	15	1	1
Weighted Sum		5.20	11.60	5.40	8.40	5.60
Final Ranking		5	13	6	10	7

Legend		
	1	Ranking =1
	2	Ranking = 2 or 3
	15	Ranking >= 4

Table 7 Overall Ranking Of Corrected Matrix Methodologies

Criteria	Weighting	Corrected R-matrix Swing Bus Methodology	Corrected R-matrix Area Load Methodology	Corrected R-matrix 50% Area Load Methodology	Corrected R-matrix Direct Methodology	Corrected R-matrix Gradient Methodology	Corrected R-matrix Gradient/2 Methodology
Shift Factor	1	6	10	1	11	8	4
Number of Generators That Exceed the Limits	1	10	8	4	6	10	4
Range of Loss Factors	1	11	8	3	6	10	4
Seasonal Volatility	1	10	8	3	5	9	4
Swing Independent	1	9	1	1	1	1	1
Weighted Sum		9.20	7.00	2.40	5.80	7.60	3.40
Final Ranking		11	8	1	6	9	2

Criteria	Weighting	Kron Matrix Direct Methodology	Kron Matrix Swing Bus Methodology	Kron Matrix Gradient/2 Methodology	Corrected R-matrix ILF Methodology	Branch Loss Matrix Flow Tracking
Shift Factor	1	5	9	3	7	1
Number of Generators That Exceed the Limits	1	1	7	3	8	2
Range of Loss Factors	1	1	7	2	9	5
Seasonal Volatility	1	1	7	2	6	11
Swing Independent	1	9	9	9	1	1
Weighted Sum		3.40	7.80	3.80	6.20	4.00
Final Ranking		2	10	4	7	5

Legend		
	1	Ranking =1
	2	Ranking = 2 or 3
	15	Ranking >= 4

Table 8 Overall Ranking Of Uncorrected Matrix Methodologies

Criteria	Weighting	Uncorrected R-matrix Swing Bus Methodology	Uncorrected R-matrix Area Load Methodology	Uncorrected R-matrix 50% Area Load Methodology	Uncorrected R-matrix Direct Methodology	Uncorrected R-matrix Gradient Methodology	Uncorrected R-matrix Gradient/2 Methodology
Shift Factor	1	4	6	2	3	5	1
Number of Generators That Exceed the Limits	1	6	4	1	2	5	3
Range of Loss Factors	1	6	4	1	3	5	2
Seasonal Volatility	1	6	4	1	3	5	2
Swing Independent	1	6	1	1	1	1	1
Weighted Sum		5.60	3.80	1.20	2.40	4.20	1.80
Final Ranking		6	4	1	3	5	2

Legend		
	1	Ranking =1
	2	Ranking = 2 or 3
	15	Ranking >= 4

Table 9 Loss Factors by Load Flow Area, 50% Area Load Corrected Matrix Methodology

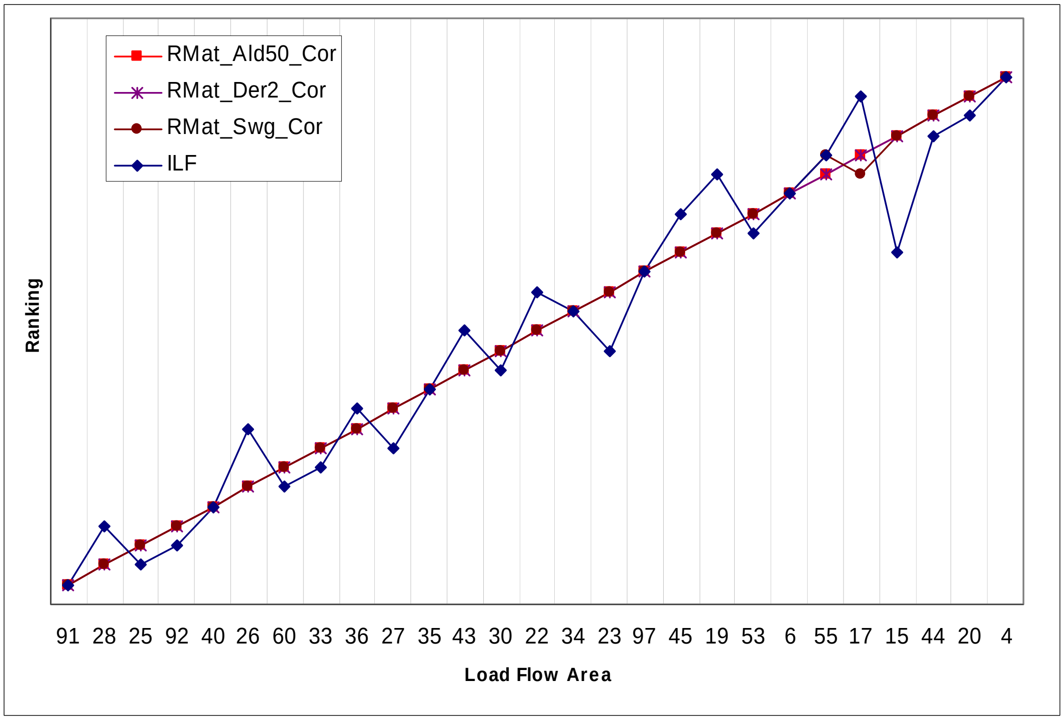
Area	Average	Maximum	Minimum	Range
4	-5.81%	-5.66%	-5.93%	0.27%
6	-3.09%	-2.73%	-3.46%	0.74%
15	-4.16%	-4.16%	-4.16%	0.00%
17	-3.90%	-3.35%	-4.05%	0.70%
19	-2.67%	-2.67%	-2.67%	0.00%
20	-4.60%	-2.57%	-7.00%	4.44%
22	0.68%	0.68%	0.68%	0.00%
23	-0.33%	0.09%	-0.54%	0.63%
25	8.92%	9.24%	8.63%	0.62%
26	4.01%	4.01%	4.01%	0.00%
27	3.26%	3.26%	3.26%	0.00%
28	9.40%	11.10%	8.26%	2.84%
30	0.88%	1.12%	0.50%	0.62%
33	3.78%	4.23%	3.04%	1.19%
34	-0.10%	0.02%	-0.22%	0.24%
35	1.59%	1.85%	1.41%	0.43%
36	3.74%	4.18%	2.92%	1.26%
40	6.11%	6.42%	5.67%	0.76%
43	1.04%	1.07%	1.02%	0.05%
44	-4.33%	-3.89%	-4.86%	0.98%
45	-2.55%	-1.81%	-3.31%	1.50%
53	-2.80%	-1.58%	-3.77%	2.19%
55	-3.89%	-3.01%	-4.77%	1.77%
60	3.94%	3.97%	3.84%	0.13%
91	10.24%	11.51%	9.98%	1.53%
92	8.88%	8.97%	8.82%	0.14%
97	-2.17%	2.33%	-4.69%	7.03%

Legend
- Maximum
- Minimum
- Range > 2% < max

Table 10 Loss Factors by Load Flow Area, 50% Area Load Uncorrected Matrix Methodology

Area	Average	Maximum	Minimum	Range
4	-7.51%	-7.31%	-7.66%	0.35%
6	-4.34%	-3.99%	-4.70%	0.70%
15	-6.41%	-6.41%	-6.41%	0.00%
17	-12.04%	-11.28%	-12.28%	1.00%
19	-5.12%	-5.12%	-5.12%	0.00%
20	-6.15%	-3.74%	-9.00%	5.26%
22	-0.10%	-0.10%	-0.10%	0.00%
23	-1.05%	-0.54%	-1.31%	0.78%
25	12.30%	12.66%	11.95%	0.70%
26	3.99%	3.99%	3.99%	0.00%
27	3.98%	3.98%	3.98%	0.00%
28	11.82%	13.84%	10.39%	3.45%
30	0.35%	0.54%	0.01%	0.53%
33	4.20%	4.53%	3.64%	0.89%
34	-0.69%	-0.57%	-0.82%	0.25%
35	0.90%	1.13%	0.75%	0.39%
36	2.77%	3.28%	1.82%	1.46%
40	5.98%	6.46%	5.15%	1.31%
43	-0.72%	-0.70%	-0.74%	0.04%
44	-5.88%	-5.12%	-6.84%	1.72%
45	-3.97%	-3.15%	-4.80%	1.65%
53	-5.00%	-3.83%	-6.06%	2.23%
55	-6.01%	-5.00%	-7.03%	2.04%
60	4.14%	4.16%	4.10%	0.06%
91	13.83%	15.89%	13.43%	2.46%
92	12.23%	12.33%	12.17%	0.16%
97	-3.83%	1.95%	-6.92%	8.88%

Legend	
	Maximum
	Minimum
	Range > 2% < max



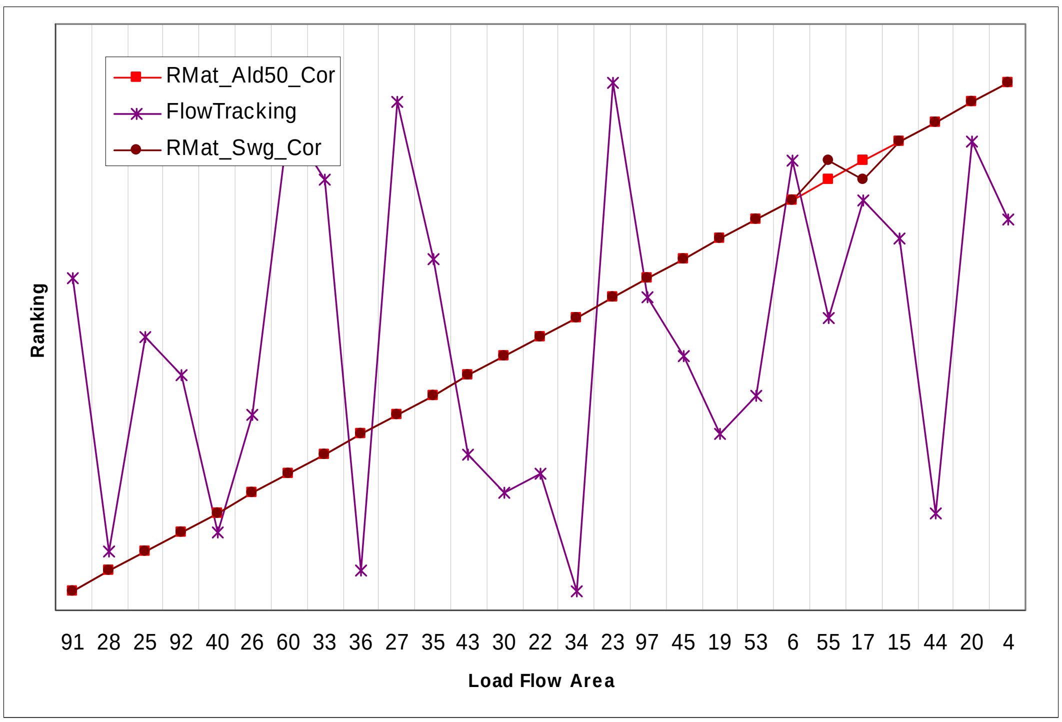


Figure 1 Ranking of Loss Factors by Load Flow Area

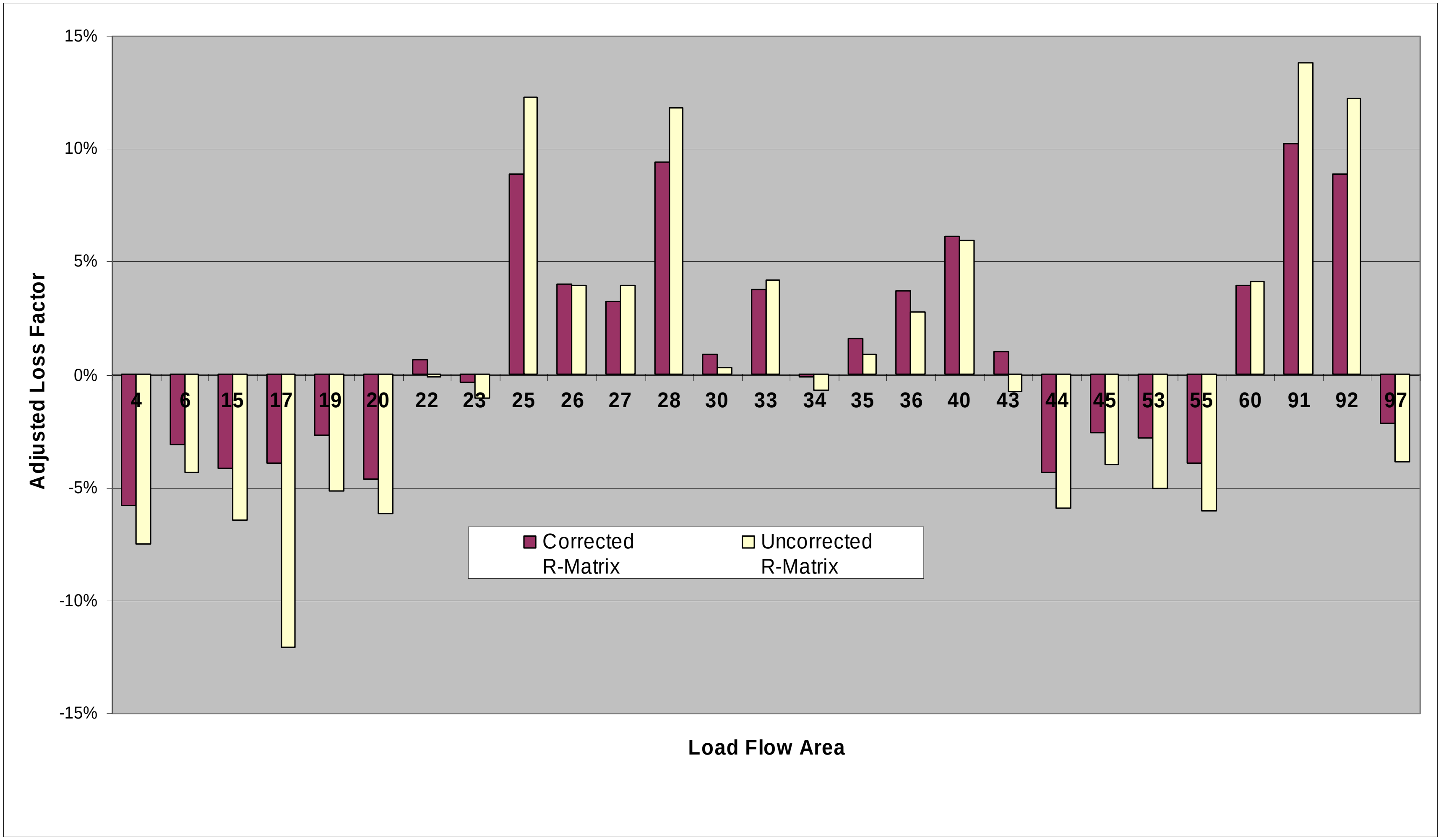


Figure 2 Comparison of Adjusted Average Loss Factors Using Corrected and Uncorrected R-Matrices

Appendix A

Derivation of 50% Area Load Adjustment Loss Factors

Appendix A

Derivation of 50% Area Load Adjustment Loss Factors

Losses ' L ' can be calculated from two simultaneous equations.

$$L = (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot (\mathbf{Pg} + \mathbf{Pl}) \qquad \text{Equation (1)}$$

$$L = \sum_i \left(\mathbf{Pg}_i + \mathbf{Pl}_i\right) \qquad \text{Equation (2)}$$

R is the real component of the inverse of the corrected symmetrical open circuit admittance matrix.

Pg is a vector of all the generator power injections at each of the nodes defined by the **R** matrix (normally positive)

Pl is a vector of power injections due to loads at each of the nodes (normally negative)

From Equation (1), the change in losses due to a change in generator output ΔPg_j at node 'j' is:

$$\Delta L_j = 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot (\mathbf{\Delta Pg}_j + \mathbf{\Delta Pl}_j) \qquad \text{Equation (3)}$$

where:

$$\mathbf{\Delta Pg}_j = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T \Delta Pg_j \qquad \text{Equation (4)}$$

and the change in distributed load to accommodate the change in generation is:

$$\mathbf{\Delta Pl}_j = \delta_j \cdot \mathbf{Pl} \qquad \text{Equation (5)}$$

where: $\delta_j \cdot$ is an adjustment factor applied to all loads due to the change in generation at node 'j'.

From Equation (2), then the change in losses is also equal to:

$$\Delta L_j = \Delta Pg_j + \delta_j \cdot \sum_i \mathbf{Pl}_i \qquad \text{Equation (6)}$$

The required load adjustment factor is therefore:

$$\delta_j = \frac{\Delta L_j - \Delta Pg_j}{\sum_i \mathbf{Pl}_i} \qquad \text{Equation (7)}$$

Substituting into Equation (3) for $\Delta \mathbf{Pg}_j$, $\Delta \mathbf{Pl}_j$, and δ_j yields:

$$\Delta L_j = 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \left(\begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T \Delta Pg_j + \left(\frac{\Delta L_j - \Delta Pg_j}{\sum_i \mathbf{Pl}_i} \right) \cdot \mathbf{Pl} \right) \qquad \text{Equation (8)}$$

Dividing by ΔPg_j

$$\frac{\Delta L_j}{\Delta Pg_j} = 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \left(\begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T + \left(\frac{\Delta L_j}{\Delta Pg_j} - 1 \right) \cdot \frac{\mathbf{Pl}}{\sum_i \mathbf{Pl}_i} \right) \qquad \text{Equation (9)}$$

Collecting terms:

$$\frac{\Delta L_j}{\Delta Pg_j} \left(1 - 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \frac{\mathbf{Pl}}{\sum_i \mathbf{Pl}_i} \right) = -2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \frac{\mathbf{Pl}}{\sum_i \mathbf{Pl}_i} + 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T \qquad \text{Equation (10)}$$

Defining:

$$\alpha = 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \frac{\mathbf{Pl}}{\sum_i \mathbf{Pl}_i} \qquad \text{Equation (11)}$$

Equation (10) can be re-written:

$$\frac{\Delta L_j}{\Delta Pg_j} (1 - \alpha) + \alpha = 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T \qquad \text{Equation (12)}$$

Considering the generation at node 1:

$$\frac{\Delta L_1}{\Delta Pg_1}(1-\alpha)+\alpha = 2\cdot(\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}}\cdot\mathbf{R}\cdot\begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}^{T}$$ Equation (13)

Multiplying by the generation at node 1, Pg_1 gives:

$$\frac{\Delta L_1}{\Delta Pg_1}(1-\alpha)\cdot Pg_1+\alpha\cdot Pg_1 = 2\cdot(\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}}\cdot\mathbf{R}\cdot\begin{bmatrix}Pg_1 & 0 & \cdots & 0\end{bmatrix}^{T}$$ Equation (14)

Similarly for generation at node 2:

$$\frac{\Delta L_2}{\Delta Pg_2}(1-\alpha)\cdot Pg_2+\alpha\cdot Pg_2 = 2\cdot(\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}}\cdot\mathbf{R}\cdot\begin{bmatrix}0 & Pg_2 & 0 & \cdots & 0\end{bmatrix}^{T}$$ Equation (15)

Summing Equations similar to (14) and (15) for all nodes:

$$(1-\alpha)\cdot\begin{bmatrix}\frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n}\end{bmatrix}\cdot\mathbf{Pg}+\alpha\cdot\sum_i \mathbf{Pg_i} = 2\cdot(\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}}\cdot\mathbf{R}\cdot\mathbf{Pg}$$ Equation (16)

Expanding the second term by substituting for α:

$$\alpha\cdot\sum_{\mathbf{i}} \mathbf{Pg_i} = 2\cdot(\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}}\cdot\mathbf{R}\cdot\frac{\mathbf{Pl}}{\sum_i \mathbf{Pl}_i}\sum_i \mathbf{Pg_i}$$ Equation (17)

Rearranging the terms:

$$\alpha\cdot\sum_{\mathbf{i}} \mathbf{Pg_i} = 2\cdot(\mathbf{Pg}+\mathbf{Pl})^{\mathbf{T}}\cdot\mathbf{R}\cdot\mathbf{Pl}\cdot\frac{\sum_i \mathbf{Pg_i}}{\sum_i \mathbf{Pl}_i}$$ Equation (18)

From Equation (2):

$$\sum_i \mathbf{Pg}_i = L-\sum_i \mathbf{Pl}_i$$ Equation (19)

Substituting into Equation (18) for $\sum_i \mathbf{Pg}_i$ gives:

$$\alpha \cdot \sum_i \mathbf{Pg_i} = 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{Pl} \cdot \left(\frac{L}{\sum_i \mathbf{Pl}_i} - 1 \right) \qquad \text{Equation (20)}$$

and substituting for $\alpha \cdot \sum_{\mathbf{i}} \mathbf{Pg_i}$ gives:

$$(1-\alpha) \cdot \begin{bmatrix} \frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n} \end{bmatrix} \cdot \mathbf{Pg} + 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{Pl} \cdot \left(\frac{L}{\sum_i \mathbf{Pl}_i} - 1 \right) = 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{Pg} \qquad \text{Equation (21)}$$

which is equal to:

$$(1-\alpha) \cdot \begin{bmatrix} \frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n} \end{bmatrix} \cdot \mathbf{Pg} + \alpha \cdot L = 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{Pg} + 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot \mathbf{Pl} \qquad \text{Equation (22)}$$

or:

$$(1-\alpha) \cdot \begin{bmatrix} \frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n} \end{bmatrix} \cdot \mathbf{Pg} + \alpha \cdot L = 2 \cdot (\mathbf{Pg} + \mathbf{Pl})^{\mathbf{T}} \cdot \mathbf{R} \cdot (\mathbf{Pg} + \mathbf{Pl}) \qquad \text{Equation (23)}$$

$$(1-\alpha) \cdot \begin{bmatrix} \frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n} \end{bmatrix} \cdot \mathbf{Pg} + \alpha \cdot L = 2L \qquad \text{Equation (24)}$$

Or, collecting terms:

$$(1-\alpha) \cdot \frac{\begin{bmatrix} \frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n} \end{bmatrix}}{2} \cdot \mathbf{Pg} = L \cdot \left(1 - \frac{\alpha}{2} \right) \qquad \text{Equation (25)}$$

Rearranging Equation (25):

$$L = \frac{(1-\alpha)}{\left(1-\frac{\alpha}{2}\right)} \cdot \frac{\begin{bmatrix} \frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n} \end{bmatrix}}{2} \cdot \mathbf{Pg} \qquad \text{Equation (26)}$$

If ' α ' is small

$$L \approx \left(1-\frac{\alpha}{2}\right) \cdot \frac{\begin{bmatrix} \frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n} \end{bmatrix}}{2} \cdot \mathbf{Pg} \qquad \text{Equation (27)}$$

If the generation vector is divided into two components representing those that pay for losses ' $\mathbf{Pg_{ass}}$ ' and those than do not ' $\mathbf{Pg_{unass}}$ ', i.e.:

$$\mathbf{Pg} = \mathbf{Pg_{ass}} + \mathbf{Pg_{unass}} \qquad \text{Equation (28)}$$

then, Equation (27) can be re-written:

$$L \approx \left(1-\frac{\alpha}{2}\right) \cdot \frac{\begin{bmatrix} \frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n} \end{bmatrix}}{2} \cdot (\mathbf{Pg_{ass}} + \mathbf{Pg_{unass}}) \qquad \text{Equation (29)}$$

or:

$$L \approx \mathbf{Lf_{raw}}^{\mathbf{T}} \cdot \mathbf{Pg_{ass}} + L_{shift} \qquad \text{Equation (30)}$$

In Equation (29), the raw loss factor ' $\mathbf{Lf_{raw}}^{\mathbf{T}}$ ' is defined to be:

$$\mathbf{Lf_{raw}} = \frac{\begin{bmatrix} \frac{\Delta L_1}{\Delta Pg_1} & \cdots & \frac{\Delta L_n}{\Delta Pg_n} \end{bmatrix}^{\mathbf{T}}}{2} \qquad \text{Equation (31)}$$

and ' L_{shift} ' represents the losses that are unassigned based on raw loss factors. I.e.:

$$L_{shift} = \left(1-\frac{\alpha}{2}\right)\frac{\left[\frac{\Delta L_1}{\Delta Pg_1} \quad \cdots \quad \frac{\Delta L_n}{\Delta Pg_n}\right]}{2}\cdot \mathbf{Pg_{unass}} - \frac{\alpha}{2}\cdot\frac{\left[\frac{\Delta L_1}{\Delta Pg_1} \quad \cdots \quad \frac{\Delta L_n}{\Delta Pg_n}\right]}{2}\cdot(\mathbf{Pg_{ass}}) \qquad \text{Equation (32)}$$

Applying a shift factor to all raw loss factors to recover the unassigned losses, Equation (30) can be re-written:

$$L \approx \mathbf{Lf_{raw}}^{\mathbf{T}} \cdot \mathbf{Pg_{ass}} + L_{shift}\cdot[1 \quad \cdots \quad 1]^{\mathbf{T}} \cdot \frac{\mathbf{Pg_{ass}}}{\sum_i \mathbf{Pg_{ass\,i}}} \qquad \text{Equation (33)}$$

from which:

$$L \approx \mathbf{Lf_{adj}}^{\mathbf{T}} \cdot \mathbf{Pg_{ass}} \qquad \text{Equation (34)}$$

where:

$$\mathbf{Lf_{adj}} = \left[\left(\tfrac{1}{2}\frac{\Delta L_1}{\Delta Pg_1} + Sf\right) \quad \cdots \quad \left(\tfrac{1}{2}\frac{\Delta L_n}{\Delta Pg_n} + Sf\right)\right]^{\mathbf{T}} \qquad \text{Equation (35)}$$

and from Equations (32) and (33):

$$Sf = \frac{\left(1-\frac{\alpha}{2}\right)\frac{\left[\frac{\Delta L_1}{\Delta Pg_1} \quad \cdots \quad \frac{\Delta L_n}{\Delta Pg_n}\right]}{2}\cdot \mathbf{Pg_{unass}} - \frac{\alpha}{2}\cdot\frac{\left[\frac{\Delta L_1}{\Delta Pg_1} \quad \cdots \quad \frac{\Delta L_n}{\Delta Pg_n}\right]}{2}\cdot(\mathbf{Pg_{ass}})}{\sum_i \mathbf{Pg_{ass\,i}}}\cdot \qquad \text{Equation (36)}$$

If from Equation (1), if it is assumed that the load and generation will contribute roughly the same order of magnitude to losses, then it can be deduced that the factor 'α' will have an order of magnitude in the range of the average loss factor for the system. With typical loss factors in the range of 5% for the system, the approximation introduced in Equation (27) above is valid.
Since the total unassigned generation is less than 1% of the total generation in the system, the component of the shift factor due to unassigned generation in Equation (36) will be less than 1% of typically 5% or less than 0.0005 (0.05%). The component of the loss factor due to

approximations in the contribution of the assigned generators will be in the order of 2½ % of 5% or about 0.00125 (0.125%).