Alberta Electric System Operator

Loss Factor Methodologies Evaluation
Part 1 - Determination of 'Raw' Loss Factors

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ALBERTA ELECTRIC SYSTEM OPERATOR

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1 INTRODUCTION

This report discusses the results of full system testing of different methodologies to develop individual generator loss factors to allocate losses to generators for a specific load flow condition.

2 EVALUATION METHODOLOGY

The full Alberta Integrated Electric System (AIES) was used as the basis for all calculations. A full set of twelve 2003 load flow conditions as used in AESO's current loss factor calculations was used as the reference power flow cases for all alterative methodologies. The load flow model consists of about 1700 busses, among which 730 have generators, loads or both connected. Bus number 1520 (the 500 kV equivalent of the BC Hydro and WECC system) was designated as the swing bus for the system.

Table 1 presents a summary of the twelve load flow solutions. With the exceptions discussed hereinafter, the summary is based on PSLF Version 13.4 accounting methods. In the load flow data, motor loads are modelled as negative generators; so, total PSLF generation reflects the net component. The contributions of the generation and motor load components have been separated out in the tabulation. The tabulation is similar to the tabulation expected from PSS/E with one exception. PSS/E treats all shunt paths as loads (including transformer no-load losses). PSLF treats transformer shunt paths as magnetizing losses; hence, their contribution to the power balance is included in the 'losses' category.

2.1 Approaches to Loss Factor Calculations

2.1.1 Direct Approach

In the direct approach, loss factors are extracted directly from matrix equations describing the relationship between system losses and generation and load at each bus. The equations are examined, and arranged in a form such as the following:

$$\begin{aligned} \text{Losses} = {} & K_1(Pg_1, Pg_2, --Pg_n, Pl_1, Pl_2, --Pl_n) \cdot Pg_1 \ldots \\ & + K_2(Pg_1, Pg_2, --Pg_n, Pl_1, Pl_2, --Pl_n) \cdot Pg_2 \ldots \\ & + \ldots \\ & + K_n(Pg_1, Pg_2, --Pg_n, Pl_1, Pl_2, --Pl_n) \cdot Pg_n \ldots \\ & + K_0 \end{aligned} \qquad \text{Equation (1)}$$

where Pg_i represents the output of generator "i" and Pl_i represents the magnitude of the load "i". In the direct method, the loss factor for generator "i" is set to the function:

$$lf_i = K_i(Pg_1, Pg_2, --Pg_n, Pl_1, Pl_2, --Pl_n) \qquad \text{Equation (2)}$$

This function which when evaluated for each generator is multiplied directly by the generator output, providing an indication of the generators contribution to total system losses. The function can therefore be equated to a loss factor.

The term K_0 in Equation (1) represents all components of the total system loss that are independent of generation. This component of the losses is not accounted for during the assignment of losses to generation and therefore will represent the contribution of the direct methodology to the shift factor required to balance the assigned loss equation.

2.1.2 Gradient Method

In the gradient method, the loss factor of a single generator is determined from its marginal impact on transmission losses. The gradient, equal to the change in system losses for a given change in individual unit generation can be calculated analytically by differentiation of Equation (1) or numerically using tools such as a load flow to make small changes to individual generator output, and monitoring the impact of the change in system losses. The raw loss factor for each generator is set equal to the gradient. The gradient method may over or under assign losses resulting in a requirement for a shift factor to balance the loss equation.

2.1.3 Gradient by 2 Method

The gradient method provides a very good estimate of the incremental losses caused by each generator. However, as losses are typically a function of the square of the generation, it does not provide a very good indication of contribution of the total output of the generator to the losses. It can be shown analytically that 100% of the losses can be attributed to both generators and loads based on ½ of their individual gradients. However, as the contributions dues to loads must be assigned to the generators, the contribution to losses can be expressed as a shift factor to each of the generator loss factors to balance the loss equation.

2.2 Calculation of Gradients

2.2.1 Present AESO Swing Bus Method

The present AESO loss factor methodology uses a single swing bus method in which one generator is designated as a swing bus and loss factors are calculated for each other generator based on load flow results. The generator loss factor is equal to the change in losses for a small change in output for the generation for which the loss factor is being calculated. By definition, the raw loss factor of the generator at the swing bus is zero.

2.2.2 Area Load Adjustment

In the area load adjustment method, the generator for which the loss factor is being determined is designated as the swing generator, and load is changed at every bus in the area by a constant ratio. For this calculation the 'area' is selected to be the entire Alberta system. Again loss factor is calculated equal to the change in losses for the resultant change in generation at the swing bus.

2.2.3 Partial Differentiation

A third method for calculating gradient based loss factors is to set the loss factor for each generator equal to the partial derivative of the loss equation with respect to the output of the generator. This is a purely mathematical expression for loss factor and there is an underlying assumption that all other contributions to the loss equation remain constant.

The loss factor for each generator based on Equation (1) would be equal to the 'direct' loss factor (i.e. the function defined by Equation (2)) plus an additional component equal to the partial derivative of the function with respect to the generator output.

2.3 Solution Methods

In the matrix analysis approaches, loss factors were determined directly from matrices describing the relationship between generator power, bus loads and ac system topology.

The matrix analysis includes an approximate (uncorrected) or exact (corrected) loss matrix describing the dependency of losses on both generation and load. In addition, loss factors were determined using the Kron loss matrix equation in which losses are expressed as only a direct function of generation.

2.4 Methodologies Evaluated

Generator loss factors were determined for each of the methodologies given in Section 3 and the results were compared as discussed in Section 4.

3 METHODOLOGIES EVALUATED

3.1 Uncorrected Loss Matrix (Direct)

In this methodology, the loss factors are determined directly from the coefficients of a system loss matrix.

The system loss matrix is derived from topology and is of the form:

$$R_{uncorr} = \left(\overline{Y^{-1}}\right)^{T} \cdot M^{T} \cdot G \cdot M \cdot Y^{-1} \qquad \text{Equation (3)}$$

where:

Y is the nodal admittance matrix for the system
$\left(\overline{Y^{-1}}\right)^{T}$ is the transpose of the conjugate of the inverse of the nodal admittance matrix
M is the branch incidence matrix
G is the diagonal matrix of branch conductances.

The uncorrected 'R' matrix is in effect the real component of the inverse of the nodal admittance matrix Y.

Losses can be calculated directly using the expression

$$Losses = \left(\bar{I}\right)^{T} \cdot R \cdot I \qquad \text{Equation (4)}$$

Where I is a vector of current injections corresponding to each generator and load bus of the system and $\left(\bar{I}\right)^{T}$ is the transpose of the conjugate of the vector of current injections.

To a first approximation, the loss equation using the system loss matrix can be written in the form:

$$Losses = \left(P_g + P_l\right)^{T} \cdot R \cdot \left(P_g + P_l\right) \qquad \text{Equation (5)}$$

where P_g contains the generator output (p.u.) and P_l contains the negative values of individual loads (p.u.). Loads are treated as negative generators in this equation.

The equation can be re-written in the form:

$$\text{Losses} = \left(P_g + 2P_l\right)^T \cdot R \cdot P_g + P_l^T \cdot R \cdot P_l \qquad \text{Equation (6)}$$

In this expression, losses can be expressed as a function of two components: one component that is independent of generation and another component that is dependent on both load and generation.

The component that is a function of generation is of the form:

$$\text{Losses}_g = \text{LossFactor} \cdot P_g \qquad \text{Equation (7)}$$

where:

$$\text{LossFactor} = \left(P_g + 2P_l\right)^T \cdot R \qquad \text{Equation (8)}$$

In this methodology, loss factors were calculated **directly** from the above Equation (8).

The loss matrices "R" used in this analysis were the 'uncorrected matrices, based only on system topology.

In this method, the losses that are a function of only the load component are the major contributor to the unassigned losses. There is an additional component due to errors in loss estimation introduced as a result of using an uncorrected loss matrix.

3.2 Corrected Loss Matrix (Direct)

If load flow information (such as bus voltages, angles and generator and load power factors) is available, each individual term of the loss matrix can be 'corrected' by the expression:

$$\zeta_{i,j} = \frac{\cos\left[\left(\phi_i - \phi_j\right) - \left(\sigma_i - \sigma_j\right)\right]}{v_i \cdot v_j \cdot \cos\left(\phi_i\right) \cdot \cos\left(\phi_j\right)} \qquad \text{Equation (9)}$$

$$R_{corr_{i,j}} = R_{uncorr_{i,j}} \cdot \zeta_{i,j} \qquad \text{Equation (10)}$$

where:

subscripts i and j point to elements of the 'R' matrix, corresponding to buses in the system.

ϕ_i and ϕ_j correspond to the net power factor angles at buses i and j respectively.

σ_i and σ_j correspond to the voltage angles at buses i and j respectively.

v_i and v_j correspond to the magnitudes of the voltages at buses i and j respectively.

With these corrections, Equation (5) above becomes an 'exact' numerical expression of losses.

In this set of calculations the corrected loss matrices were used. Corrections were based on bus voltages, bus angles and generator and load power factors obtained from the base-case load flow solutions.

With the corrected loss matrix, Equation (5) above gives exactly the same numerical value for total system losses as the load flow.

3.3 Swing Bus Methodology Using Uncorrected Loss Matrix

Equation (5) above can be used to determine the change in losses for a small change in swing bus and loss factor bus generation.

It can be shown that if the loads are unchanged, the change in total system losses due to change in generation is approximately given by:

$$\Delta\text{Losses} = 2\left(P_g + P_l\right)^T \cdot R \cdot \Delta P_g \qquad \text{Equation (10)}$$

It is also known that the change in losses is equal to the sum of the change in losses in all generators, i.e.:

$$\Delta\text{Losses} = \sum_i \Delta P_{g_i} \qquad \text{Equation (11)}$$

If generation is assumed to be constant at all but the swing bus and the bus at which the loss factor is being calaulated, the above equations reduce to two equations in three unknowns (Δlosses, ΔP_{g1}, ΔP_{gi})

The simultaneous equations can be combined to calculate the ratio:

$$\frac{\Delta\text{Losses}}{\Delta P_{g_i}}$$

which is effectively the definition of raw loss factors used in the present AESO loss factor methodology.

For these calculations, the bus 493 (Clover Bar) was used as the swing bus for the calculations. This is consistent with the present AESO swing bus loss factor methodology.

3.4 Swing Bus Methodology Using Corrected Loss Matrix

The calculation discussed in 3.3 above was repeated using the corrected loss matrix. In using the 'corrected loss matrix for this calculation, the set of assumptions change. For the uncorrected loss matrix calculations, it is mathematically exact to assume that the 'R' matrix does not change with small changes in load, as the uncorrected 'R' matrix is a function of only system topology. Assuming the corrected 'R' matrix to be constant implies that all of the corrections made to the 'R' matrix are also independent of small changes in generation.

In practice, a small change in generator power output is not likely to significantly alter bus voltages. Load power factors will remain constant, in the same manner as a load flow solution. Generator power factors however are likely to change particularly at the generator where the loss factor is being evaluated and the swing bus. Assuming a constant power factor could lead to undesired consequences.

Any generator operating with a low power factor (for example units connected primarily for var support) would be very susceptible to high loss factor calculations. Assuming the power factor to be constant implies that with every increment in generator output there is a corresponding increase in generator var output. As actual transmission losses are not only a function of MW but also Mvar, the small change in generator output could have a significant impact on total system losses associated with the assumption of a constant 'R' matrix. The net result is that low power factor generators could be assessed excessively large loss factor penalties or credits.

A second undesirable effect of this assumption is that some generators could be penalized in terms of increased loss factors for supplying vars to the system under conditions when vars are needed on the system. It is also conceivable that some generators and associated loads could receive credits for taking vars from the system under var shortage conditions.

One method of circumventing this issue is to treat all var injections, from both loads and generators as equivalent constant admittance shunt devices. The nodal admittance matrix must be adjusted to include this effect, before the 'R' matrix is established.

The implication of this treatment of load and generator vars is that the load and generator var injections are treated as being constant. Since bus voltages are assumed to be constant, the vars generated by the equivalent shunt devices are also constant. This is again a reasonable approximation for small changes in generator output.

If the power market evolves to include equivalent var loss factors for both generators and loads, these assumptions would need to be revisited.

3.5 Area Load Methodology Using Uncorrected Loss Matrix

Equation (5) above can be also be used to determine the change in losses for a small change in swing bus generation and total system load. If all of the loads in the system are increased by a small percentage (say δ), the total change in system losses can be shown to be approximated by the following expression:

$$\Delta \text{Losses} = 2\left(P_g + P_l\right)^T \cdot R \cdot \Delta P_g + \delta \cdot 2 \cdot \left(P_g + P_l\right)^T \cdot R \cdot P_l \qquad \text{Equation (12)}$$

$$\Delta \text{Losses} = \sum_i \Delta P_{g_i} + \delta \cdot \sum_j P_{l_j} \qquad \text{Equation (13)}$$

If only the generation at the loss factor bus changes, then again the above equations can be reduced to two simultaneous equations in three unknowns (Δlosses, ΔP_{g1}, δ)

The simultaneous equations can be combined to again calculate the ratio:

$$\frac{\Delta \text{Losses}}{\Delta P_{g_i}}$$

For this methodology, the generator for which the loss factor is calculated effectively becomes the swing machine for the system. Hence the loss factors calculated are independent of an arbitrary selection of a swing bus in the system.

3.6 Area Load Methodology Using Corrected Loss Matrix

The calculation method discussed in 3.5 above was repeated using the corrected loss matrix. This method is again subject to the limitations introduced by the assumptions regarding the constant 'R' matrix discussed in Section 3.4. Generator and load vars are treated as equivalent shunt devices and hence are indirectly assumed to be constant, by the assumption of constant voltages.

As the main objective of loss factors is to define the relationship between generator power output and transmission losses, it is reasonable to assume that the variation in system load is related only to the power component, i.e., the change in load vars is zero. The assumption of constant load vars in this 'corrected' 'R' matrix methodology is therefore reasonable.

3.7 Uncorrected Loss Matrix (Gradient Method)

The partial derivative of equation 5 above with respect to individual generator output can be determined for each generator as follows:

$$\frac{\partial}{\partial P_{g_i}}(\text{Losses}) = 2\cdot\left(P_g + P_l\right)^T\cdot R\cdot S(i) \qquad \text{Equation (14)}$$

where S(i) is a vector in which the i^{th} element is 1.0 and all other elements are zero.

A vector of all the gradients is simply:

$$\frac{\partial}{\partial P_g}(\text{Losses}) = 2\cdot\left(P_g + P_l\right)^T\cdot R \qquad \text{Equation (15)}$$

The above can be used to allocate losses to generators by multiplying each individual gradient by the generator output.

3.8 Corrected Loss Matrix (Gradient Method)

The calculation discussed in 3.7 above can be repeated using the corrected loss matrix. Again the loss factors are dependent on the assumption of a constant 'R" matrix. This is a mathematically exact assumption, however the impacts of the assumption are the same as discussed in Section 3.4. Load and generator var outputs must be assumed to be constant and be embedded in the 'R' matrix to avoid unrealistic penalties and credits for vars supplied or absorbed from the system.

3.9 Uncorrected Loss Matrix (Gradient by 2 Method)

If Equation (15) above is expanded to included all buses for which generation or load is included, it can be combined with Equation (2) to give:

$$\text{Losses} = \left[\frac{\frac{\partial}{\partial P_g}(\text{Losses})}{2}\right]\cdot\left(P_g + P_l\right) \qquad \text{Equation (16)}$$

I.e. the total losses of the system can be allocated to load and generation buses based on ½ the gradient calculated for each generator and load bus. The component that is due to generation can be determined from:

$$\text{Losses}_g = \left[\frac{\frac{\partial}{\partial P_g}(\text{Losses})}{2}\right]\cdot P_g \qquad \text{Equation (17)}$$

and the component of the losses due to load is given by:

$$\text{Losses}_{l} = \left[\frac{\frac{\partial}{\partial P_{g}}(\text{Losses})}{2} \right] \cdot P_{l}$$ Equation (18)

The term $\left[\frac{\frac{\partial}{\partial P_{g}}(\text{Losses})}{2} \right]$ in Equation (17) can be considered to be a vector of generator raw loss factors and the term "Losses_{l}" of Equation (18) can be considered to be unassigned losses that are due to loads and which must be factored into the loss balance equation using a shift factor.

One advantage of this methodology is that there is a quantitative explanation of all components of the losses.

3.10 Corrected Loss Matrix (Gradient by 2 Method)

The calculation discussed in 3.9 above can be repeated using the corrected loss matrix. Again the assumption regarding the constant 'R' matrix discussed herein is applicable.

3.11 50% Area Load Methodology, Uncorrected Loss Matrix

It will be shown that the losses assigned by the area load adjustment methodology are almost twice the actual losses. The loss factors calculated using area load adjustment could be reduced by 50% and unassigned losses and shift factor recalculated.

3.12 50% Area Load Methodology, Corrected Loss Matrix

The loss factors calculated using area load adjustment and the corrected loss matrix can also be reduced by 50% and unassigned losses and shift factor recalculated. It will be shown that the unassigned losses and resultant shift factor for this methodology are essentially zero.

Again the assumption regarding the constant 'R' matrix discussed herein is applicable.

3.13 Kron Loss Matrix (Direct Methodology)

An alternative matrix expression of losses used for optimal power flow solutions is the Kron loss matrix formula.

The equation is of the form:

$$\text{Losses} = P_{g}^{T} \cdot B02 \cdot P_{g} + B01 \cdot P_{g} + B00$$ Equation (19)

In the above equation, P_g is a vector housing the magnitude of the real output of the generators. B02 is a matrix, B01 is a vector and B00 is a simple scalar.

The loss equation above can be rewritten in the form:

$$\text{Losses} = \left(P_g^{\,T} \cdot B02 + B01 \right) \cdot P_g + B00 \qquad \text{Equation (20)}$$

The bracketed term "$\left(2 \cdot P_g^{\,T} \cdot B02 + B01 \right)$" can be considered to be a vector of raw loss factors as it allocates all but the component "B00" of the losses to the generators. The term B00 represents an unallocated loss component that will contribute to the shift factor.

3.14 Kron Loss Matrix (Swing Bus Methodology)

The Kron loss equation can be rearranged in a similar fashion to the loss matrix equation to determine loss factors based on the existing swing bus methodology.

$$\Delta \text{Losses} = 2 \cdot \left(P_g^T \cdot B02 + B01 \right) \cdot \Delta P_g \qquad \text{Equation (21)}$$

It is also known that the change in losses is equal to the sum of the change in losses in all generators, i.e.:

$$\Delta \text{Losses} = \sum_j \Delta P_{g_j} \qquad \text{Equation (22)}$$

If generation is assumed to be constant at all but the swing bus and loss factor bus, the above equations reduce to two equations in three unknowns (ΔLosses, ΔP_{g1}, ΔP_{gi})

The simultaneous equations can be combined to calculate the ratio:

$$\frac{\Delta \text{Losses}}{\Delta P_{g_i}} \qquad \text{Equation (23)}$$

This is effectively the definition of raw loss factors used in the present AESO methodology.

Similar to the corrected loss matrix methods discussed above, this method assumes that the coefficients B02, B01 and B00 are constant. While the coefficients are not as straight forward as the loss matrix 'R' matrix calculations, imbedded in the formulation of the coefficients are corrections for bus voltages, power factors and power angles. As a result, the implications of the assumption of constant coefficients in this methodology are the same as the assumption of constant 'R' matrix in the corrected loss matrix methodologies.

3.15 Kron Loss Matrix (Gradient by 2 Method)

The partial derivative of Equation (19) above can be determined for each generator as follows:

$$\frac{\partial}{\partial P_{g_i}}(\text{Losses}) = \left(2 \cdot P_g{}^T \cdot B02 + B01\right) \cdot S(i) \qquad \text{Equation (24)}$$

where S(i) again is a vector in which the i^{th} element is 1.0 and all other elements are zero.

The vector

$$G = 2 \cdot P_g{}^T \cdot B02 + B01 \qquad \text{Equation (25)}$$

therefore contains all of the gradients calculated for each generator.

If the gradient is dominated by the first term in Equation (25), the loss equation can be approximated by:

$$\text{Losses} = \frac{G}{2} \cdot P_g + B00 + \varepsilon \qquad \text{Equation (26)}$$

where the term ε represents the error introduced by the approximation by ignoring the B01 component and which must be compensated for in the shift factor along with the B00 term.

Again the coefficients B02, B01 and B00 area all assumed to be constant in this methodology. Similar to the loss matrix methodologies discussed in Section 3.8 and 3.10, this is mathematically correct but the implications are the same as discussed in 3.14 above.

4 COMPARISON OF METHODOLOGIES

Loss factors were calculated for every generator in the Alberta system for each of the twelve 2003 base-case load flows and for each of the 15 methodologies discussed in Section 3 above. The results of these calculations are summarized herein.

4.1 Required Shift Factor

Table 2 and Table 3 summarize the shift factors associated with each load flow and each methodology. The magnitude of the shift factor is a measure of the ability of each methodology to allocate total system losses on a mathematically defined basis. In this context, shift factor is defined to be the correction that must be made to the loss factor for each individual generator to account for all of the unassigned power (MW) losses in the system. A positive shift factor implies that the methodology would result in an under-assignment of total system losses. I.e., the loss factors of each generator must be increased by the shift factor to recover all of the power flow losses. A negative shift factor implies an over-assignment of losses.

The column "Average Loss Factor" is the ratio of losses to total generation as calculated using a load flow program.

The seasonal average shift factors are simply the average of the shift factors for the three load flows of each season. The annual average shift factor is the average of the four seasonal shift factors (equivalent to the average of the shift factors for all 12 load flows). The average shift factors have no physical interpretation, but are useful for comparing the methodologies.

The shift factors shown in Table 2 and Table 3 are the same. In Table 2, the largest and smallest magnitude shift factors encountered for each methodology, for each power flow, are highlighted. In Table 3 the loss factors for each load flow are compared. The largest and smallest magnitude shift factors encountered for each methodology on a load flow basis are highlighted.

Table 2 indicates that there is no apparent correlation between shift factors and load flow or season. For example, the largest shift factor does not always occur for a specific season or load flow condition, independent of methodology. For some methodologies the largest shift factor occurs under winter peak conditions but for others the smallest shift factor occurs for that load flow condition.

Table 3 however does start to indicate a trend in results. The 50% area load adjustment methodologies (both corrected and uncorrected matrices) account for all of the smallest shift factors calculated. The largest shift factors occur with the following methodologies:

- uncorrected R matrix, area load adjustment
- corrected R matrix, Direct methodology
- Kron Matrix, Swing bus methodology

The corrected and uncorrected loss matrix swing bus methodologies require similar shift factors. Both under-allocate losses, and both require shift factors similar in magnitude the current AESO swing bus methodology.

The corrected and uncorrected loss matrix area load adjustments again require similar shift factors. Both over-allocate losses. In fact, both methods over allocate by an amount that is almost equal to the average loss factor, particularly for the corrected loss matrix methodology.

If the loss factors computed with this method are reduced by a factor of 2, resulting in loss factors that are 50% of the area load adjustment methodology, the required shift factor as indicated above is extremely small.

The shift factors required for the uncorrected and corrected loss matrix direct methodologies are not similar in magnitude. This indicates that the methodology is extremely sensitive to assumptions made in the creation of the loss matrix. Both approaches under-assign losses but the shift factors required for the corrected matrix methodology are actually greater than the average system loss factor implying that the total losses accounted for by the methodology are negative.

Similar to the load area adjustment methodology, the loss matrix gradient method significantly over-assigns losses. The corresponding methodology with ½ gradients under-assigns losses, but in this case, the shift factor calculated using the corrected loss matrix is actually greater that the shift factor calculated using an uncorrected matrix. In the corrected matrix, the shift factor is due entirely to the contribution of the system loads to the losses. In the uncorrected method, inaccuracies introduced by the uncorrected loss matrix tend to counteract the effects of the loads. This would not occur if system voltage profiles were lower.

The direct and gradient by 2 Kron matrix based methodologies slightly under-assign losses with the gradient by 2 methodology requiring the lowest shift factor. The Kron matrix swing bus methodology shows less consistent results between load flows. The Kron matrix methodology is being investigated further to assess the cause of the inconsistencies.

4.2 Range of Loss Factors

The Alberta Department of Energy has indicated that all 'normalized' loss factors must be no greater than twice the average system loss factor and no less than the negative value of the average system loss factor

The range of loss factors after application of the shift factors described in Section 4.1 provides an indication of the extent that loss factors calculated using each methodology would exceed the Department's requirements. Table 4 summarizes the variations in loss factors that could be expected and provides an indication of the degree of ultimate loss factor correction that eventually would have to be applied.

In the table, the "maximum loss factor" is the largest adjusted seasonal loss factor (12 case average) based on individual generators. "Minimum loss factor" is the smallest (or largest negative) value and "range of loss factors" is the difference between the two extremes.

The table also indicates the number of generators with loss factors greater than the criteria and the number of generators with loss factors less than the criteria along with the total. Although the loss factors on which the table is based have been adjusted to take into account and balance all of the power flow losses, an additional correction would be required to take into account differences between load flow losses and forecast generator volumes and losses. The next level of correction will shift the range and as a result, the number of generators with loss factors greater than the maximum permitted may change (say increase), but the number of generators with loss factors less than criteria will also change (i.e. decrease) but the change in total number of generators exceeding the criteria should not be significant.

The Kron matrix direct methodology has the lowest range of loss factors and as a result also has the least number of loss factors that exceed the criteria. The uncorrected loss matrix swing-bus methodology has the largest range and consequently the largest number of generators (86) exceeding the criteria.

4.3 Seasonal Volatility

The Alberta Department of Energy has also indicated that each generator will be assigned a single loss factor. This loss factor will represent the contribution of the generator to losses on an annual basis (at minimum). As the loss factors will be based on some average (weighted or un-weighted) of loss factors calculated using load flows as a starting point, the seasonal volatility of the loss factor becomes an indicator of the degree of accuracy that can be expected when assigning energy based loss factors.

Table 4 also indicates the seasonal volatility of loss factors for each methodology. Volatility is expressed as the largest range in individual generator loss factors over each of the four seasons.

Loss factors calculated using the Kron matrix direct and gradient by 2 methodologies are least sensitive to the variations introduced by the four seasons. This is followed closely by the 50% area load adjustment methodologies and the loss matrix gradient by 2 methods. The range in seasonal volatility for these six methods is from 4.01 to 5.1%.

The uncorrected loss matrix swing bus methodology has the largest seasonal volatility at 11.45%.

4.4 Ranking of Alternative Methodologies.

Each of the methodologies has certain advantages and disadvantages. To quantify the overall assessment of the methodologies, a ranking has been determined for each of the metrics.

The first metric assessed was the load flow adjustment shift factor. Table 3 indicated that the magnitude of shift factor was dependent on not only the methodology but also the individual load flow condition and the season. To assess this metric, the methodologies were ranked for each load flow condition from 1 to 15, depending on the magnitude of load flow shift factor as shown in Table 5. The methodologies were also ranked in terms of the seasonal and annual loss factors from 1 to 15. A ranking of 1 indicates the most desirable while a ranking of 15 is least desirable.

A weighted average of each of the individual rankings was determined for each methodology. The weightings assigned were:

Individual load flows	1/36
Individual Seasons	3/36
Annual Shift Factor	12/36

The weightings effectively give equal weight (1/3) to all of the load flows, all of the seasons and the annual shift factor.

Table 5 indicates that the methodology with the lowest ranking or minimum overall shift factor is the corrected loss matrix, 50% area load adjustment methodology. The methodology with the highest weighted average is the loss matrix direct methodology. The methodologies have been ranked from 1 to 15 based on the weighted average of the individual ranking as shown in the table.

The methodologies have also been ranked from 1 to 15 based on each of the other metrics discussed above. These are:

- The number of generators that exceed the loss factor limits
- The range of loss factors
- Seasonal Volatility

A fifth metric also considered was the dependency of the methodology on selection of swing bus. A problem associated with those methodologies that are dependent on the selection of swing bus for the system is actually designating the appropriate swing. The most appropriate swing bus may need to change with changes in topology and system loading conditions. Those methodologies with no dependence on swing bus selection were assigned a rank of 1 (all tied for 1st place). Those methodologies where there is a dependence on swing bus selection were assigned a ranking of 15 (tied for last place).

Each of the metric rankings were assigned an equal weighting and a weighted sum factoring all of the metrics was determined. The methodology for ranking of alternatives is shown in Table 6. The ranking is based on the weighted sum of the individual rankings.

Based on this assessment method, both loss matrix area load adjustment methodologies rank in the top two, with the corrected l loss matrix method on top followed by the uncorrected loss matrix method.

The direct and gradient by 2 methodologies based on the Kron matrix formula are ranked the same in position 5.

The loss matrix swing bus methodologies are ranked last.

As the methodologies can be separated into two distinct groups, namely those base on corrected matrices and those based on uncorrected matrices, the ranking process describe above was repeated for each group. The comparable rankings are shown in Table 7 and Table 8.

The 50% area load adjustment methodology remains at the top in both categories. The Kron loss formula based methods improve to positions 3 and 4 in the corrected matrix grouping with the direct methodology in position 3. The area load swing bus methodology remains in last place in all groupings.

4.5 Recommendation

Based on the rankings of alternatives, it is clear that the loss matrix 50% area load adjustment methodology is the best approach to allocating losses to generators. The methodology results in a small load flow shift factor. Generator loss factors are independent of the selection of the swing bus for the system. I.e. when the loss factor is calculated for each generator, the bus to which the generator is connected must become the swing bus for the system. The number of generators that are likely to drive loss factor compression is small (in the order of 12) and the extent of compression required is low with a requirement to reduce the loss factor range from about 18.5% to three times the average loss factor or about 15%.

One of the other requirements of the Alberta Department of Energy is that with the chosen methodology, loss factors of nearby (electrical) generators be similar.

Loss factors were calculated for each generator in each of the load flow areas. The results are given in Table 9 for the corrected loss matrix and Table 10 for the uncorrected loss matrix, 50% area load adjustment methodologies. In Table 9, the variation in adjusted loss factors varies from as low as 0.05% in load flow area 43 (Sheerness) to as high as 7.03% in load flow area 97 (designated as "IPP site"). The variation in area 40 (Lake Wabamun accounting for the majority of the Alberta system generation) is only 0.76%.

Although there is a slight shift in the loss factors within each area when calculated with the uncorrected loss matrix the range remains about the same, in particular in area 40 where the range of loss factor variation remains low at 1.31%.

A comparison of the average loss factors for each of the load flow areas and for both the corrected and un-corrected loss matrices is given in Figure 1. The pattern evident in the average loss factors for each load flow area for the uncorrected matrix methodology is similar to the corresponding pattern with the corrected matrix methodology. However, the loss factors (both penalties and credits) are sufficiently different so as to limit the usefulness of the uncorrected matrix methodology.

The uncorrected matrix methodology has advantages in terms of transparency. These methodologies eliminate the variation introduced into the loss factor calculation as a result of load flow solution.

For the existing methodology, loss factors for all new generators are based on information deemed to be confidential by the generators. This information is embedded in the load flows and as a result, the load flows themselves have also been deemed to be confidential. If the loss factor calculations were based on an uncorrected loss matrix, the calculation would be dependent only on system topology and assumed distribution of generation and loads. System topology and data is openly available through TASMO. The distribution of loads is not considered confidential and the stacking order for generation is public information. The only unavailable quantity would be the amount of generation assumed for each entry in the stacking order as this information is considered to be confidential. It should be possible, however, to establish a reasonable estimate of the generation distribution based on historical system performance and posted representative system load flows.

If an approach to loss factor calculations is adopted that is based on historical utilization of the transmission system by each generator, the confidentiality issue may disappear, and all aspects of the loss factor calculations could become public.

In this case, the value of the uncorrected matrix methodologies diminishes. The corrected matrix methodology should be adopted because of its more accurate distribution of load flow losses.

The recommended methodology therefore for determining load flow based 'raw' loss factors is the corrected loss matrix, 50 % area load adjustment methodology.

Table 1 Load Flow Solution Summary

	WnPk	WnMd	WnLw	SpPk	SpMd	SpLw	SmPk	SmMd	SmLw	FlPk	FlMd	FlLw
Total Generation	**8456.8**	**7845.7**	**7548.6**	**7978.7**	**7554.2**	**7297.3**	**8269.2**	**7594.3**	**7331.2**	**8390.3**	**7737.9**	**7459.0**
Generation	8423.8	7812.7	7515.6	7945.7	7521.2	7264.3	8236.2	7561.3	7298.2	8357.3	7704.9	7426.0
Negative loads	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0
Total Imports	**258.9**	**-8.3**	**-604.1**	**98.0**	**-70.3**	**-673.3**	**94.6**	**-33.1**	**-706.8**	**433.4**	**3.6**	**-619.1**
SPC Imports	100.0	-0.1	-75.0	100.0	0.0	-75.0	100.0	-0.1	-75.0	100.0	0.0	-75.0
BC Imports	158.9	-8.2	-529.1	-2.0	-70.3	-598.3	-5.4	-33.0	-631.8	333.4	3.6	-544.1
Total Loads	**8345.3**	**7473.8**	**6536.0**	**7718.0**	**7144.5**	**6236.6**	**8020.0**	**7229.3**	**6236.2**	**8468.4**	**7393.4**	**6449.7**
Constant P Loads	8043.6	7172.4	6234.4	7453.2	6879.6	5971.6	7725.8	6935.1	5942.0	8173.4	7098.4	6154.7
Motor Loads	276.2	276.2	276.2	239.4	239.4	239.4	294.2	294.2	294.2	295.0	295.0	295.0
Shunts	25.5	25.2	25.3	25.4	25.4	25.6	0.0	0.0	0.0	0.0	0.0	0.0
Load Flow Losses	**370.5**	**363.5**	**408.5**	**358.7**	**339.5**	**387.4**	**343.8**	**331.9**	**388.2**	**355.3**	**348.1**	**390.2**
Generation + imports less loads	370.5	363.5	408.5	358.7	339.5	387.4	343.8	331.9	388.2	355.3	348.1	390.2
Mismatch	**0.000**	**0.001**	**0.039**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**

Table 2 Load Flow Shift Factors Required For Each Methodology (Part "a")

Loading Condition	Average Loss Factor	Uncorrected R-matrix Swing Bus Methodology	Corrected R-matrix Swing Bus Methodology	Uncorrected R-matrix Area Load Methodology	Corrected R-matrix Area Load Methodology	Uncorrected R-matrix 50% Area Load Methodology	Corrected R-matrix 50% Area Load Methodology	Uncorrected R-matrix Direct Methodology	Corrected R-matrix Direct Methodology
WnPk	4.77%	2.07%	1.43%	-4.79%	-4.57%	-0.01%	0.10%	2.07%	7.87%
WnMd	5.16%	3.75%	2.88%	-5.25%	-4.99%	-0.04%	0.09%	1.73%	7.47%
WnLw	6.42%	4.19%	3.99%	-7.82%	-6.37%	-0.70%	0.02%	0.86%	6.58%
SpPk	5.01%	2.06%	1.48%	-4.93%	-4.84%	0.04%	0.09%	1.91%	7.87%
SpMd	5.05%	3.30%	2.43%	-5.09%	-4.90%	-0.02%	0.07%	1.63%	8.21%
SpLw	6.41%	3.38%	3.37%	-7.66%	-6.47%	-0.62%	-0.03%	0.93%	6.85%
SmPk	4.32%	1.69%	1.20%	-4.80%	-4.15%	-0.24%	0.08%	1.79%	6.67%
SmMd	4.55%	3.44%	2.67%	-5.12%	-4.42%	-0.29%	0.06%	1.34%	6.43%
SmLw	6.03%	3.04%	3.43%	-8.02%	-6.05%	-0.99%	-0.01%	0.57%	5.93%
FlPk	4.22%	1.03%	0.58%	-4.50%	-4.06%	-0.14%	0.08%	0.57%	6.26%
FlMd	4.65%	3.70%	2.93%	-5.36%	-4.53%	-0.35%	0.06%	1.30%	5.64%
FlLw	5.86%	3.24%	3.42%	-7.70%	-5.86%	-0.92%	0.00%	0.74%	5.55%
Winter Average		3.34%	2.77%	-5.95%	-5.31%	-0.25%	0.07%	1.55%	7.31%
Spring Average		2.91%	2.43%	-5.90%	-5.40%	-0.20%	0.04%	1.49%	7.64%
Summer Average		2.72%	2.43%	-5.98%	-4.88%	-0.51%	0.04%	1.23%	6.34%
Fall Average		2.66%	2.31%	-5.85%	-4.82%	-0.47%	0.05%	0.87%	5.82%
Annual Average		2.91%	2.48%	-5.92%	-5.10%	-0.36%	0.05%	1.29%	6.78%

Loading Condition	Average Loss Factor	Uncorrected R-matrix Gradient Methodology	Corrected R-matrix Gradient Methodology	Uncorrected R-matrix Gradient/2 Methodology	Corrected R-matrix Gradient/2 Methodology	Kron Matrix Direct Methodology	Kron Matrix Swing Bus Methodology	Kron Matrix Gradient/2 Methodology
WnPk	4.77%	-2.84%	-0.70%	0.96%	2.03%	1.19%	-11.95%	0.65%
WnMd	5.16%	-3.62%	-1.84%	0.77%	1.66%	1.28%	-7.63%	0.68%
WnLw	6.42%	-6.77%	-5.72%	-0.18%	0.35%	1.82%	2.50%	0.40%
SpPk	5.01%	-3.19%	-1.67%	0.91%	1.67%	1.26%	-9.02%	0.70%
SpMd	5.05%	-3.66%	-2.16%	0.69%	1.45%	1.27%	-5.68%	0.75%
SpLw	6.41%	-6.87%	-7.16%	-0.23%	-0.37%	1.98%	9.49%	0.38%
SmPk	4.32%	-2.95%	-0.43%	0.68%	1.94%	0.91%	-11.19%	0.49%
SmMd	4.55%	-3.66%	-1.85%	0.44%	1.35%	0.90%	-5.28%	0.47%
SmLw	6.03%	-7.14%	-6.41%	-0.55%	-0.19%	1.72%	5.81%	0.06%
FlPk	4.22%	-2.61%	-0.51%	0.81%	1.86%	0.86%	-12.00%	0.46%
FlMd	4.65%	-3.81%	-2.09%	0.42%	1.28%	0.89%	-5.52%	0.39%
FlLw	5.86%	-6.77%	-5.73%	-0.45%	0.07%	1.47%	3.33%	0.11%
Winter Average		-4.41%	-2.75%	0.52%	1.35%	1.43%	-5.69%	0.58%
Spring Average		-4.57%	-3.66%	0.46%	0.92%	1.50%	-1.74%	0.61%
Summer Average		-4.58%	-2.90%	0.19%	1.03%	1.18%	-3.55%	0.34%
Fall Average		-4.40%	-2.78%	0.26%	1.07%	1.07%	-4.73%	0.32%
Annual Average		-4.49%	-3.02%	0.36%	1.09%	1.29%	-3.93%	0.46%

Legend
Largest Shift Factor per Methodology
Smallest Shift Factor per Methodology

Table 3 Load Flow Shift Factors Required For Each Methodology (Part "b")

		Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix
Loading Condition	Average Loss Factor	Swing Bus Methodology	Swing Bus Methodology	Area Load Methodology	Area Load Methodology	50% Area Load Methodology	50% Area Load Methodology	Direct Methodology	Direct Methodology
WnPk	4.77%	2.07%	1.43%	-4.79%	-4.57%	-0.01%	0.10%	2.07%	7.87%
WnMd	5.16%	3.75%	2.88%	-5.25%	-4.99%	-0.04%	0.09%	1.73%	7.47%
WnLw	6.42%	4.19%	3.99%	-7.82%	-6.37%	-0.70%	0.02%	0.86%	6.58%
SpPk	5.01%	2.06%	1.48%	-4.93%	-4.84%	0.04%	0.09%	1.91%	7.87%
SpMd	5.05%	3.30%	2.43%	-5.09%	-4.90%	-0.02%	0.07%	1.63%	8.21%
SpLw	6.41%	3.38%	3.37%	-7.66%	-6.47%	-0.62%	-0.03%	0.93%	6.85%
SmPk	4.32%	1.69%	1.20%	-4.80%	-4.15%	-0.24%	0.08%	1.79%	6.67%
SmMd	4.55%	3.44%	2.67%	-5.12%	-4.42%	-0.29%	0.06%	1.34%	6.43%
SmLw	6.03%	3.04%	3.43%	-8.02%	-6.05%	-0.99%	-0.01%	0.57%	5.93%
FlPk	4.22%	1.03%	0.58%	-4.50%	-4.06%	-0.14%	0.08%	0.57%	6.26%
FlMd	4.65%	3.70%	2.93%	-5.36%	-4.53%	-0.35%	0.06%	1.30%	5.64%
FlLw	5.86%	3.24%	3.42%	-7.70%	-5.86%	-0.92%	0.00%	0.74%	5.55%
Winter Average		3.34%	2.77%	-5.95%	-5.31%	-0.25%	0.07%	1.55%	7.31%
Spring Average		2.91%	2.43%	-5.90%	-5.40%	-0.20%	0.04%	1.49%	7.64%
Summer Average		2.72%	2.43%	-5.98%	-4.88%	-0.51%	0.04%	1.23%	6.34%
Fall Average		2.66%	2.31%	-5.85%	-4.82%	-0.47%	0.05%	0.87%	5.82%
Annual Average		2.91%	2.48%	-5.92%	-5.10%	-0.36%	0.05%	1.29%	6.78%

		Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Kron Matrix	Kron Matrix	Kron Matrix
Loading Condition	Average Loss Factor	Gradient Methodology	Gradient Methodology	Gradient/2 Methodology	Gradient/2 Methodology	Direct Methodology	Swing Bus Methodology	Gradient/2 Methodology
WnPk	4.77%	-2.84%	-0.70%	0.96%	2.03%	1.19%	-11.95%	0.65%
WnMd	5.16%	-3.62%	-1.84%	0.77%	1.66%	1.28%	-7.63%	0.68%
WnLw	6.42%	-6.77%	-5.72%	-0.18%	0.35%	1.82%	2.50%	0.40%
SpPk	5.01%	-3.19%	-1.67%	0.91%	1.67%	1.26%	-9.02%	0.70%
SpMd	5.05%	-3.66%	-2.16%	0.69%	1.45%	1.27%	-5.68%	0.75%
SpLw	6.41%	-6.87%	-7.16%	-0.23%	-0.37%	1.98%	9.49%	0.38%
SmPk	4.32%	-2.95%	-0.43%	0.68%	1.94%	0.91%	-11.19%	0.49%
SmMd	4.55%	-3.66%	-1.85%	0.44%	1.35%	0.90%	-5.28%	0.47%
SmLw	6.03%	-7.14%	-6.41%	-0.55%	-0.19%	1.72%	5.81%	0.06%
FlPk	4.22%	-2.61%	-0.51%	0.81%	1.86%	0.86%	-12.00%	0.46%
FlMd	4.65%	-3.81%	-2.09%	0.42%	1.28%	0.89%	-5.52%	0.39%
FlLw	5.86%	-6.77%	-5.73%	-0.45%	0.07%	1.47%	3.33%	0.11%
Winter Average		-4.41%	-2.75%	0.52%	1.35%	1.43%	-5.69%	0.58%
Spring Average		-4.57%	-3.66%	0.46%	0.92%	1.50%	-1.74%	0.61%
Summer Average		-4.58%	-2.90%	0.19%	1.03%	1.18%	-3.55%	0.34%
Fall Average		-4.40%	-2.78%	0.26%	1.07%	1.07%	-4.73%	0.32%
Annual Average		-4.49%	-3.02%	0.36%	1.09%	1.29%	-3.93%	0.46%

Legend
Largest Shift Factor Per Load Flow or Season
Smallest Shift Factor Per Load Flow or Season

Table 4 Range of Loss Factors per Methodology

	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix
	Swing Bus Methodology	Swing Bus Methodology	Area Load Methodology	Area Load Methodology	50% Area Load Methodology	50% Area Load Methodology	Direct Methodology	Direct Methodology
Maximum Loss Factor	28.72%	18.88%	26.57%	17.82%	15.89%	11.51%	16.15%	7.95%
Minimum Loss Factor	-33.14%	-21.29%	-29.76%	-19.21%	-12.28%	-7.00%	-18.13%	-24.16%
Range of Loss Factors	61.86%	40.17%	56.33%	37.03%	28.17%	18.52%	34.28%	32.12%
No. Greater Than Maximum Permitted	20	20	20	20	19	3	17	0
No. Less Than Minimum Permitted	66	60	63	58	38	9	41	19
No of Generators Exceeding Criteria	86	80	83	78	57	12	58	19
Seasonal Volatility	11.45%	11.37%	10.22%	10.31%	4.87%	4.92%	8.07%	6.78%

	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Kron Matrix	Kron Matrix	Kron Matrix
	Gradient Methodology	Gradient Methodology	Gradient/2 Methodology	Gradient/2 Methodology	Direct Methodology	Swing Bus Methodology	Gradient/2 Methodology
Maximum Loss Factor	26.91%	18.12%	16.06%	11.66%	10.33%	17.29%	11.23%
Minimum Loss Factor	-30.34%	-19.77%	-12.57%	-7.28%	-5.30%	-18.06%	-6.35%
Range of Loss Factors	57.25%	37.89%	28.62%	18.95%	15.62%	35.35%	17.57%
No. Greater Than Maximum Permitted	20	20	20	3	0	20	3
No. Less Than Minimum Permitted	64	60	40	9	1	57	2
No of Generators Exceeding Criteria	84	80	60	12	1	77	5
Seasonal Volatility	10.43%	10.69%	4.98%	5.10%	4.01%	9.02%	4.46%

Legend
Largest Magnitude per Methodolgy
Smallest Magnitude per Methodology

Table 5 Ranking of Methodologies Based on Magnitude of Shift Factor

		Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix
Loading Condition	Average Loss Factor	Swing Bus Methodology	Swing Bus Methodology	Area Load Methodology	Area Load Methodology	50% Area Load Methodology	50% Area Load Methodology	Direct Methodology	Direct Methodology
WnPk	4.77%	10	7	13	12	1	2	9	14
WnMd	5.16%	11	9	13	12	1	2	7	14
WnLw	6.42%	10	9	15	12	5	1	6	13
SpPk	5.01%	10	6	13	12	1	2	9	14
SpMd	5.05%	10	9	13	12	1	2	7	15
SpLw	6.41%	9	8	14	10	5	1	6	11
SmPk	4.32%	8	7	13	12	2	1	9	14
SmMd	4.55%	10	9	13	12	2	1	6	15
SmLw	6.03%	8	9	15	12	6	1	5	11
FlPk	4.22%	9	6	13	12	2	1	5	14
FlMd	4.65%	10	9	13	12	2	1	7	15
FlLw	5.86%	8	10	15	13	6	1	5	11
Winter Average		10	9	14	12	2	1	7	15
Spring Average		10	9	14	13	2	1	6	15
Summer Average		9	8	14	13	4	1	7	15
Fall Average		9	8	15	13	4	1	5	14
Annual Average		9	8	14	13	3	1	6	15
Weighted Average		9.31	8.22	13.94	12.56	2.94	1.11	6.33	14.39
Overall Ranking		9	8	14	13	3	1	6	15

		Uncorrected R-matrix	Corrected R-matrix	Uncorrected R-matrix	Corrected R-matrix	Kron Matrix	Kron Matrix	Kron Matrix
Loading Condition	Average Loss Factor	Gradient Methodology	Gradient Methodology	Gradient/2 Methodology	Gradient/2 Methodology	Direct Methodology	Swing Bus Methodology	Gradient/2 Methodology
WnPk	4.77%	11	4	5	8	6	15	3
WnMd	5.16%	10	8	4	6	5	15	3
WnLw	6.42%	14	11	2	3	7	8	4
SpPk	5.01%	11	7	4	8	5	15	3
SpMd	5.05%	11	8	3	6	5	14	4
SpLw	6.41%	12	13	2	3	7	15	4
SmPk	4.32%	11	3	5	10	6	15	4
SmMd	4.55%	11	8	3	7	5	14	4
SmLw	6.03%	14	13	4	3	7	10	2
FlPk	4.22%	11	4	7	10	8	15	3
FlMd	4.65%	11	8	4	6	5	14	3
FlLw	5.86%	14	12	4	2	7	9	3
Winter Average		11	8	3	5	6	13	4
Spring Average		12	11	3	5	7	8	4
Summer Average		12	10	2	5	6	11	3
Fall Average		11	10	2	6	7	12	3
Annual Average		12	10	2	5	7	11	4
Weighted Average		11.75	9.33	2.81	5.42	6.53	11.75	3.61
Overall Ranking		11	10	2	5	7	11	4

Legend
Largest Ranking Per Load Flow or Season
Smallest Ranking Per Load Flow or Season

Table 6 Overall Ranking Of Methodologies

Criteria	Weighting	Uncorrected R-matrix Swing Bus Methodology	Corrected R-matrix Swing Bus Methodology	Uncorrected R-matrix Area Load Methodology	Corrected R-matrix Area Load Methodology	Uncorrected R-matrix 50% Area Load Methodology	Corrected R-matrix 50% Area Load Methodology	Uncorrected R-matrix Direct Methodology	Corrected R-matrix Direct Methodology
Shift Factor	1	9	8	14	13	3	1	6	15
Number of Generators That Exceed the Limits	1	15	11	13	10	6	3	7	5
Range of Loss Factors	1	15	12	13	10	5	3	8	7
Seasonal Volatility	1	15	14	10	11	3	4	8	7
Swing Independent	1	15	15	1	1	1	1	1	1
Weighted Sum		13.80	12.00	10.20	9.00	3.60	2.40	6.00	7.00
Final Ranking		15	14	11	9	2	1	7	8

Legend		
	1	Ranking =1
	2	Ranking = 2 or 3
	15	Ranking >= 4

Criteria	Weighting	Uncorrected R-matrix Gradient Methodology	Corrected R-matrix Gradient Methodology	Uncorrected R-matrix Gradient/2 Methodology	Corrected R-matrix Gradient/2 Methodology	Kron Matrix Direct Methodology	Kron Matrix Swing Bus Methodology	Kron Matrix Gradient/2 Methodology
Shift Factor	1	11	10	2	5	7	11	4
Number of Generators That Exceed the Limits	1	14	11	8	3	1	9	2
Range of Loss Factors	1	14	11	6	4	1	9	2
Seasonal Volatility	1	12	13	5	6	1	9	2
Swing Independent	1	1	1	1	1	15	15	15
Weighted Sum		10.40	9.20	4.40	3.80	5.00	10.60	5.00
Final Ranking		12	10	4	3	5	13	5

Legend		
	1	Ranking =1
	2	Ranking = 2 or 3
	15	Ranking >= 4

Table 7 Overall Ranking Of Corrected Matrix Methodologies

		Corrected R-matrix	Corrected R-matrix	Corrected R-matrix	Corrected R-matrix	Corrected R-matrix	Corrected R-matrix	Kron Matrix	Kron Matrix	Kron Matrix
Criteria	Weighting	Swing Bus Methodology	Area Load Methodology	50% Area Load Methodology	Direct Methodology	Gradient Methodology	Gradient/2 Methodology	Direct Methodology	Swing Bus Methodology	Gradient/2 Methodology
Shift Factor	1	5	8	1	9	6	3	4	7	2
Number of Generators That Exceed the Limits	1	8	7	3	5	8	3	1	6	2
Range of Loss Factors	1	9	7	3	5	8	4	1	6	2
Seasonal Volatility	1	9	7	3	5	8	4	1	6	2
Swing Independent	1	9	1	1	1	1	1	9	9	9
Weighted Sum		8.00	6.00	2.20	5.00	6.20	3.00	3.20	6.80	3.40
Final Ranking		9	6	1	5	7	2	3	8	4

Legend		
	1	Ranking =1
	2	Ranking = 2 or 3
	15	Ranking >= 4

Table 8 Overall Ranking Of Uncorrected Matrix Methodologies

		Uncorrected R-matrix	Uncorrected R-matrix	Uncorrected R-matrix	Uncorrected R-matrix	Uncorrected R-matrix	Uncorrected R-matrix
Criteria	Weighting	Swing Bus Methodology	Area Load Methodology	50% Area Load Methodology	Direct Methodology	Gradient Methodology	Gradient/2 Methodology
Shift Factor	1	4	6	2	3	5	1
Number of Generators That Exceed the Limits	1	6	4	1	2	5	3
Range of Loss Factors	1	6	4	1	3	5	2
Seasonal Volatility	1	6	4	1	3	5	2
Swing Independent	1	6	1	1	1	1	1
Weighted Sum		5.60	3.80	1.20	2.40	4.20	1.80
Final Ranking		6	4	1	3	5	2

Legend		
	1	Ranking =1
	2	Ranking = 2 or 3
	15	Ranking >= 4

Table 9 Loss Factors by Load Flow Area, 50% Area Load Corrected Matrix Methodology

Area	Average	Maximum	Minimum	Range
4	-5.81%	-5.66%	-5.93%	0.27%
6	-3.09%	-2.73%	-3.46%	0.74%
15	-4.16%	-4.16%	-4.16%	0.00%
17	-3.90%	-3.35%	-4.05%	0.70%
19	-2.67%	-2.67%	-2.67%	0.00%
20	-4.60%	-2.57%	-7.00%	4.44%
22	0.68%	0.68%	0.68%	0.00%
23	-0.33%	0.09%	-0.54%	0.63%
25	8.92%	9.24%	8.63%	0.62%
26	4.01%	4.01%	4.01%	0.00%
27	3.26%	3.26%	3.26%	0.00%
28	9.40%	11.10%	8.26%	2.84%
30	0.88%	1.12%	0.50%	0.62%
33	3.78%	4.23%	3.04%	1.19%
34	-0.10%	0.02%	-0.22%	0.24%
35	1.59%	1.85%	1.41%	0.43%
36	3.74%	4.18%	2.92%	1.26%
40	6.11%	6.42%	5.67%	0.76%
43	1.04%	1.07%	1.02%	0.05%
44	-4.33%	-3.89%	-4.86%	0.98%
45	-2.55%	-1.81%	-3.31%	1.50%
53	-2.80%	-1.58%	-3.77%	2.19%
55	-3.89%	-3.01%	-4.77%	1.77%
60	3.94%	3.97%	3.84%	0.13%
91	10.24%	11.51%	9.98%	1.53%
92	8.88%	8.97%	8.82%	0.14%
97	-2.17%	2.33%	-4.69%	7.03%

Legend	
	Maximum
	Minimum
	Range > 2% < max

Table 10 Loss Factors by Load Flow Area, 50% Area Load Uncorrected Matrix Methodology

Area	Average	Maximum	Minimum	Range
4	-7.51%	-7.31%	-7.66%	0.35%
6	-4.34%	-3.99%	-4.70%	0.70%
15	-6.41%	-6.41%	-6.41%	0.00%
17	-12.04%	-11.28%	-12.28%	1.00%
19	-5.12%	-5.12%	-5.12%	0.00%
20	-6.15%	-3.74%	-9.00%	5.26%
22	-0.10%	-0.10%	-0.10%	0.00%
23	-1.05%	-0.54%	-1.31%	0.78%
25	12.30%	12.66%	11.95%	0.70%
26	3.99%	3.99%	3.99%	0.00%
27	3.98%	3.98%	3.98%	0.00%
28	11.82%	13.84%	10.39%	3.45%
30	0.35%	0.54%	0.01%	0.53%
33	4.20%	4.53%	3.64%	0.89%
34	-0.69%	-0.57%	-0.82%	0.25%
35	0.90%	1.13%	0.75%	0.39%
36	2.77%	3.28%	1.82%	1.46%
40	5.98%	6.46%	5.15%	1.31%
43	-0.72%	-0.70%	-0.74%	0.04%
44	-5.88%	-5.12%	-6.84%	1.72%
45	-3.97%	-3.15%	-4.80%	1.65%
53	-5.00%	-3.83%	-6.06%	2.23%
55	-6.01%	-5.00%	-7.03%	2.04%
60	4.14%	4.16%	4.10%	0.06%
91	13.83%	15.89%	13.43%	2.46%
92	12.23%	12.33%	12.17%	0.16%
97	-3.83%	1.95%	-6.92%	8.88%

Legend	
	Maximum
	Minimum
	Range > 2% < max

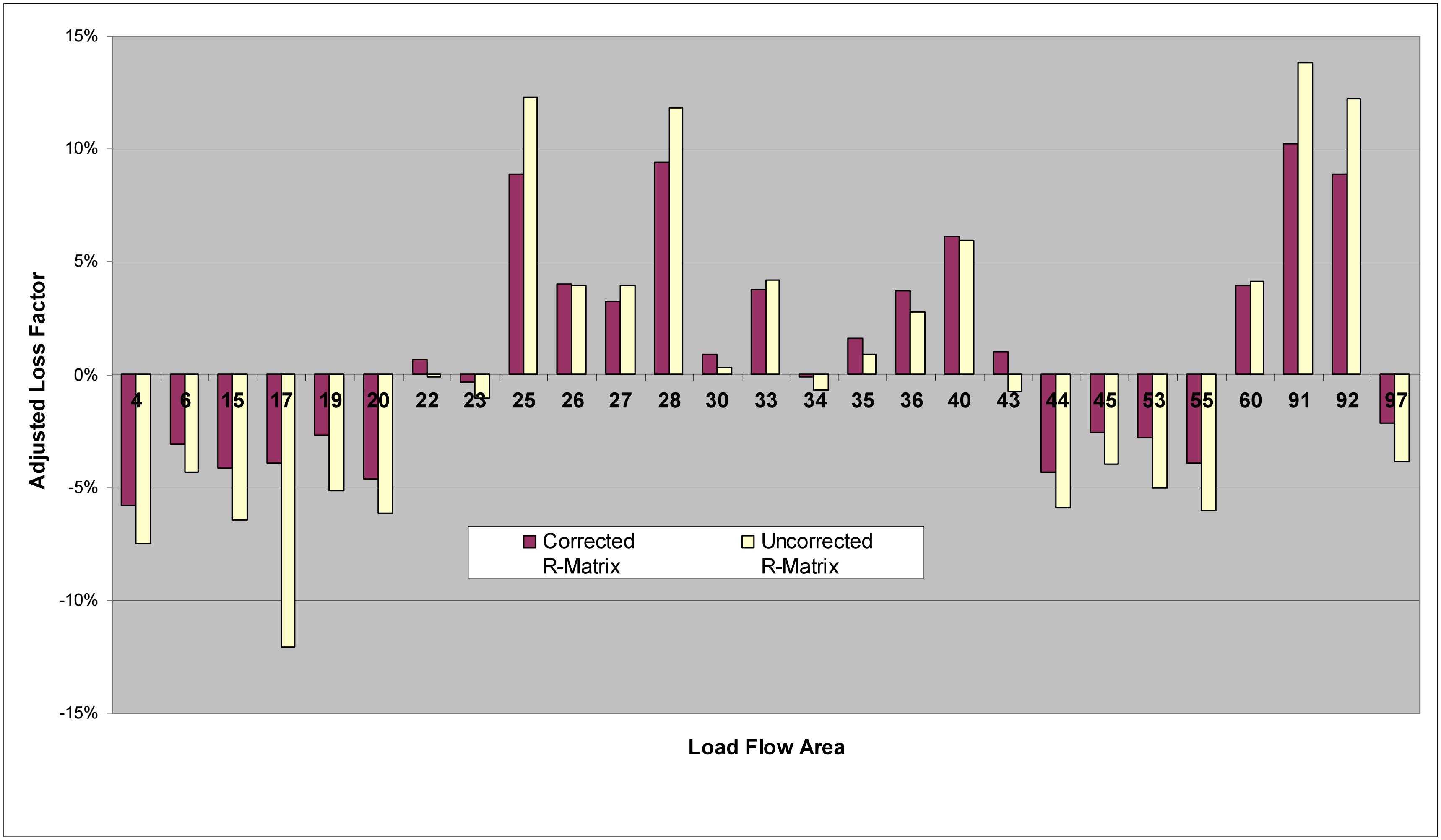

Figure 1 Comparison of Adjusted Average Loss Factors Using Corrected and Uncorrected R-Matrices